Q1.(10=3+3+2+2) An N-turn loop inductor with a radius a, made of a thin conducting wire and is connected to a resistor R as shown in the figure. Assume no losses in the loop wire. A magnetic field is present given as:

$$\mathbf{B} = B_0(3\mathbf{a}_y + 5\mathbf{a}_z)\sin(\omega t)$$

(a) Find the magnetic flux Φ linking each turn of the inductor.

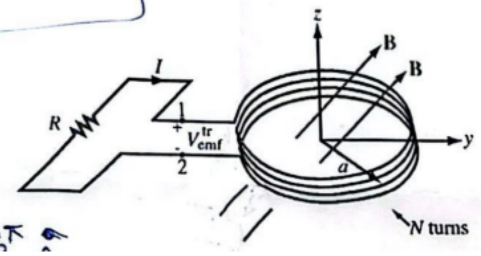
(b) The transformer emf on the terminals of the resistor R at time $t = 10$ ms, given that $N = 4$, $B_0 = 0.1$ T, $a = 5$ cm, and $\omega = 10^4$ rad/s.

(c) The induced current in the circuit for $R = 1\ \text{k}\Omega$ at $t = 10$ ms, ignoring the connecting wire resistance.

(d) What is the transformer emf of the same loop of (b) and time if the magnetic field is $\mathbf{B} = B_0\mathbf{a}_y\cos(\omega t)$.

d) $V_{emf} = 0$ V

$\psi = \int$



ⓐ $\psi = \iint B \cdot ds$

$ds = \vec{a_z}\, ds$ since the loop is in the x-y plane, then ds is in the az direction

$\psi = \iint B_0(3\vec{a_y} + 5\vec{a_z}) \cdot \vec{a_z}\ \sin(\omega t)\, ds$

$3\,\vec{a_y} \cdot \vec{a_z} = 0$, $5\,\vec{a_z} \cdot \vec{a_z} = 5$

$\psi = \iint B_0\, 5\, \sin(\omega t)\, ds = 5B_0 \sin(\omega t) \iint ds$

$\iint ds = \text{Area} = \pi a^2$

$\psi = 5 B_0 \pi a^2 \sin(\omega t)$

ⓑ $V_{emf} = -N \dfrac{\partial \psi}{\partial t}$

$\dfrac{\partial \psi}{\partial t} = 5 B_0 \pi a^2 \omega \cos(\omega t)$

$V_{emf} = -N(5B_0 \pi a^2\ \omega \cos(\omega t))$

Substituting in using values given

$V_{emf} = -135.5$ V

(C) $I = \frac{V}{R} = \frac{-135.5}{1000} = -135 \text{ mA}$

(D) when $B = B_o \cos(wt)\,\vec{a_y}$, $ds = \vec{a_z}\, ds$

$$\psi = \iint B \cdot ds = \iint B_o \cos(wt)\,\vec{a_y} \cdot \vec{a_z}\, ds = 0$$