ch 1

" vecter

1- vecter between Two points:-

$A\ (x_A,\ y_A,\ z_A)$

$B\ (x_B,\ y_B,\ z_B)$

$$\vec{AB} = B - A = \langle x_B - x_A,\ y_B - y_A,\ z_B - z_A \rangle$$

2- distance between two points:-

$$|\vec{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

3- Vecter opretions :-

م.عبدالله/0501427704

الاوراق خاصة بالمشتركين لا احلل نشرها

$$\vec{A} = A_x\ \hat{a_x} + A_y\ \hat{a_y} + A_z\ \hat{a_z}$$

$$\vec{B} = B_x\ \hat{a_x} + B_y\ \hat{a_y} + B_z\ \hat{a_z}$$

$$|\vec{A}| = \text{magnitude} = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{u_A} = \text{unite vecter} = \vec{a_A} = \frac{\vec{A}}{|\vec{A}|}$$

$$\vec{A} = 2\,\hat{a_x} + 5\,\hat{a_y} + 6\,\hat{a_z} = |\vec{A}| = \sqrt{2^2 + 5^2 + 6^2}$$

$$\vec{A} \pm \vec{B} = (A_x \pm B_x)\,\hat{a}_x + (A_y \pm B_y)\,\hat{a}_y + (A_z \pm B_z)\,\hat{a}_z$$

- Dot product → الناتج مقدار

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$$

م.عبدالله/0501427704

الاوراق خاصة بالمشتركين لا احلل نشرها

$$\vec{A} \cdot \vec{B} = 0 \qquad \theta = 90^\circ$$

perpendicular

مقدار

$$\text{comp}\ A_B = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$$

متجه

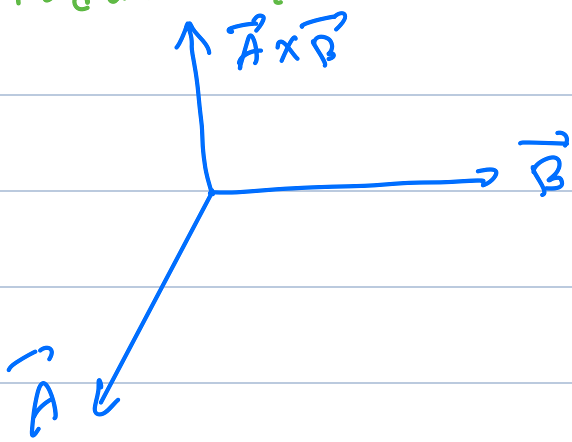
$$\text{proj}\ A_B = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}\ \vec{u}_B$$

$$\text{comp}\ B_A = \frac{\vec{B} \cdot \vec{A}}{|\vec{A}|}$$

$$\text{proj}\ B_A = \frac{\vec{B} \cdot \vec{A}}{|\vec{B}|\,|\vec{A}|}\ \vec{u}_A$$

$$\text{proj}\ A_B + \text{Normal comp}\ A_B = \vec{A}$$

- cross - product :-



$$\vec{A} \times \vec{B} = \begin{vmatrix} \overset{+}{\hat{a}_x} & \overset{-}{\hat{a}_y} & \overset{+}{\hat{a}_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - B_y A_z)\,\hat{a}_x - (A_x B_z - B_x A_z)\,\hat{a}_y + (A_x B_y - A_y B_x)\,\hat{a}_z$$

$$\sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}|\,|\vec{B}|}$$

$\vec{A} \times \vec{B} = 0 \qquad \theta = 0$

short distance

$$d = \frac{|\vec{A} \times \vec{B}|}{|B|}$$

م.عبدالله/0501427704

الاوراق خاصة بالمشتركين لا احلل نشرها

Q_1

$$\vec{A} = 6\,\hat{a_x} + 2\,\hat{a_y} - 3\,\hat{a_z}$$

$$\vec{B} = 3\,\hat{a_x} - \hat{a_y}$$

$$\overrightarrow{comp_{AB}} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} \qquad \overrightarrow{comp_{AB}}$$

$$\vec{A} \cdot \vec{B} = (6 \cdot 3) + (2 \cdot -1) + (-3 \cdot 0)$$

$$= 18 - 2 + 0 = 16$$

$$|\vec{B}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

$$\overrightarrow{comp_{AB}} = \frac{16}{\sqrt{10}} \quad \vec{U_B}$$

$$\vec{U_B} = \frac{3\,\hat{a_x} - \hat{a_y}}{\sqrt{10}}$$

$$\overrightarrow{comp_{AB}} = \frac{16}{\sqrt{10}} \cdot \frac{3\,\hat{a_x} - \hat{a_y}}{\sqrt{10}}$$

$$= \frac{16 \times 3}{10}\,\hat{a_x} - \frac{16}{10}\,\hat{a_y}$$

CH 2

"cordinates system"

1- cordinates system :-

- cartizain :-

  (x, y, z)

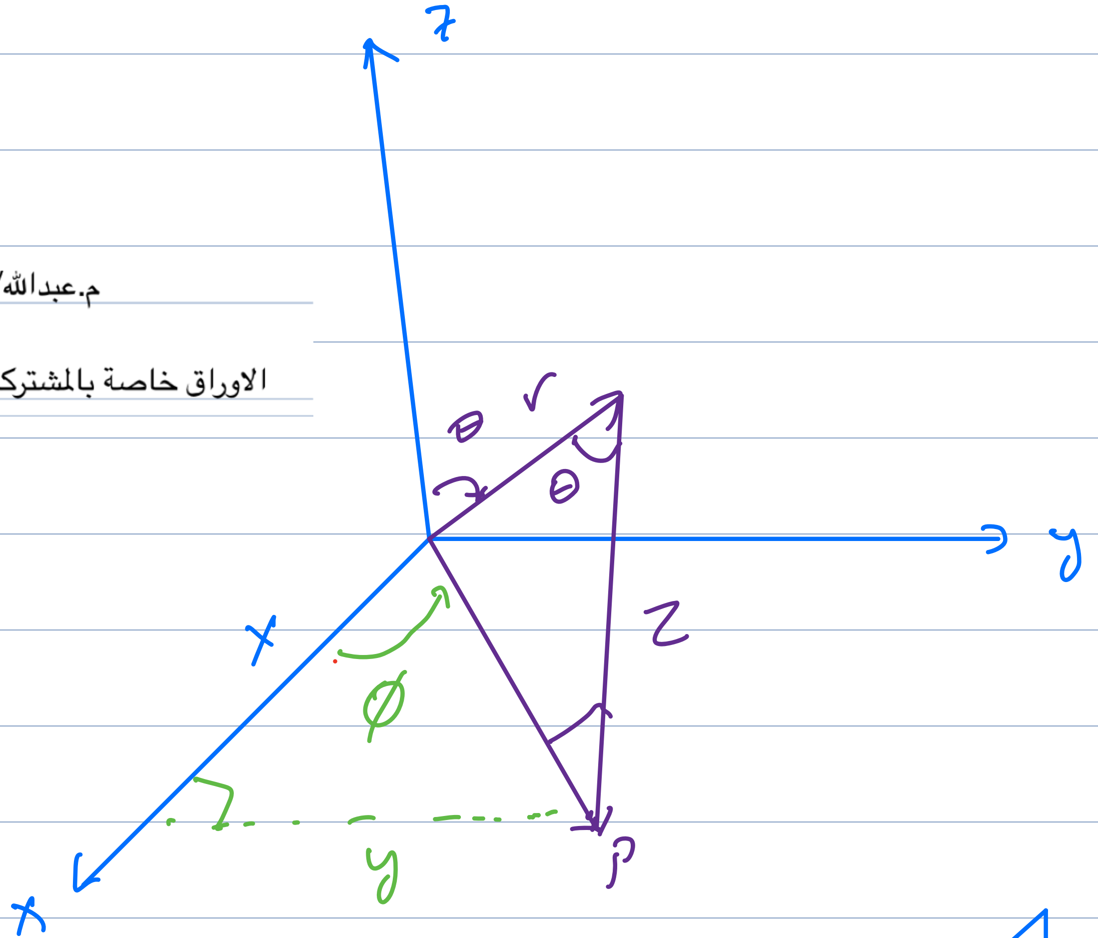
- clynider :-

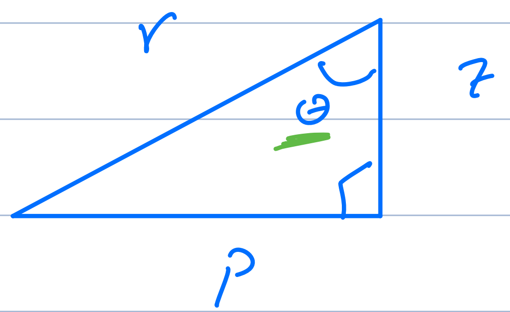
  (ρ, ϕ, z)

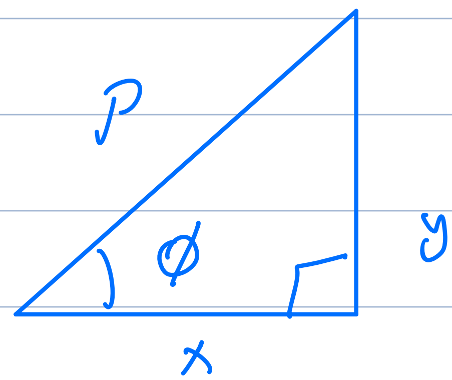
- sherical :-

  (r, θ, ϕ)

م.عبدالله/0501427704







$$r^2 = \rho^2 + z^2$$

$$r^2 = x^2 + y^2 + z^2$$

$$\rho = r \sin\theta$$

$$z = r\cos\theta$$

$$\theta = \tan^{-1}\frac{\rho}{z}$$

$$x^2 + y^2 = \rho^2$$

$$\cos\phi = \frac{x}{\rho}$$

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$\phi = \tan^{-1}\frac{y}{x}$$

2- Change between cordinate:-

point → القوانين الى فوق

scaler → ″ ″

vector → المصفوفات الي تحت

م.عبدالله/0501427704

الاوراق خاصة بالمشتركين لا احلل نشرها

Cart ⇄ cly

	A_ρ	A_ϕ	A_z
A_x	$\cos\phi$	$\sin\phi$	0
A_y	$\sin\phi$	$\cos\phi$	0
A_z	0	0	1

$$A_\rho = A_x \cos\phi + A_y \sin\phi + 0\, A_z$$

$$A_x = \cos\phi\, A_\rho + A_\phi \sin\phi + 0\, A_z$$

$$A_\phi = A_x \sin\phi + A_y \cos\phi + A_z\, 0$$

$A_z = \cancel{A_r \cos\theta}_{0} - A_\theta \sin\theta + \cancel{A_\phi}(0)_{0}$

$A_z = -A_\theta \sin\theta$

cart $\rightarrow$ sph
$\leftarrow$

	A_r	A_θ	A_ϕ
A_x	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
A_y	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
A_z	$\underline{\cos\theta}$	$-\sin\theta$	0

cly $\rightarrow$ sph
$\leftarrow$

	A_r	A_θ	A_ϕ
A_ρ	$\sin\theta$	$\cos\theta$	0
A_ϕ	0	0	1
A_z	$\cos\theta$	$-\sin\theta$	0

م.عبدالله/0501427704

Ex:-

Points $(3, 4, 5)$ find cly and sphere

$\downarrow x \quad \downarrow y \quad \downarrow z$

cly (ρ, ϕ, z)

sph (r, θ, ϕ)

$\rho = \sqrt{x^2+y^2} = 5$

$\phi = \tan^{-1}\frac{y}{x} = 53{,}13^\circ$

$z = 5$

cly $(5, 53{,}13^\circ, 5)$

$r = \sqrt{x^2+y^2+z^2} = 5\sqrt{2}$

$\theta = \tan^{-1}\frac{\rho}{z} = 45^\circ$

sph $(5\sqrt{2}, 45^\circ, 53{,}13^\circ)$

Ex:-

$V = 6\rho z^2$

$V = 6\sqrt{x^2+y^2}\, z^2$ cart

$V = 6\, r\sin\theta\,(r\cos\theta)^2$ sph

Ex:-

$$\vec{E} = xy\,\hat{a_x} + 2z\,\hat{a_y} + 2y\,\hat{a_z}$$

at $(0, 1, 1)$

$$\vec{E} = \underbrace{2\,\hat{a_y}}_{\vec{E_\rho}} + \underbrace{2\,\hat{a_z}}_{\vec{F_z}}$$

$$\phi = \tan^{-1}\frac{y}{x} = 90^\circ$$

	A_ρ	A_ϕ	A_z
A_x	$\cos\phi$	$\sin\phi$	0
A_y	$\sin\phi$	$\cos\phi$	0
A_z	0	0	1

$$\vec{E_\rho} = \underbrace{\vec{E_x}}_{0}\,\cancel{\cos\phi} + \vec{E_y}\sin\phi + \underbrace{\vec{E_z}}_{}\cancel{(0)}$$

$$\vec{E_\rho} = \vec{E_y}\sin\phi = 2\sin(90^\circ) = 2$$

$$\vec{E_\phi} = \vec{E_y}\cos\phi = 2\cos(90^\circ) = 0$$

$$\vec{E_z} = 2$$

$$\vec{E}_{cly} = 2\,\hat{a_\rho} + 2\,\hat{a_z}$$

$$\vec{E}_{sph} = \frac{4}{\sqrt{2}}\,\hat{a_r}$$ Try it

CH 3

"Vecter calculas"

1- length:- $\vec{dl}$

$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$\vec{dl} = d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

$$\vec{dl} = dr\,\hat{a}_r + \boxed{r}\,\boxed{d\theta}\,\hat{a}_\theta + \boxed{r\sin\theta}\,\boxed{d\phi}\,\hat{a}_\phi$$

2- Area:-

$$\vec{ds} = dx\,dy\,\hat{a}_z = dy\,dz\,\hat{a}_x = dx\,dz\,\hat{a}_y$$

$$\vec{ds} = \rho\,d\phi\,dz\,\hat{a}_\rho = d\rho\,dz\,\hat{a}_\phi = \rho\,d\phi\,d\rho\,\hat{a}_z$$

$$\vec{ds} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r = r\,dr\,d\theta\,\hat{a}_\phi$$

$$= r\sin\theta\,dr\,d\phi\,\hat{a}_\theta$$

3- Volume :-

$$\vec{dv} = dx\, dy\, dz$$

$$\vec{dv} = \rho\, d\rho\, d\phi\, dz$$

$$\vec{dv} = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

4- Gradiant :- $\nabla v \longrightarrow$ الناتج متجه

$$\nabla v = \frac{\partial v}{\partial x}\hat{a_x} + \frac{\partial v}{\partial y}\hat{a_y} + \frac{\partial v}{\partial z}\hat{a_z}$$

$$\nabla v = \frac{\partial v}{\partial \rho}\hat{a_\rho} + \frac{1}{\rho}\cdot\frac{\partial v}{\partial \phi}\hat{a_\phi} + \frac{\partial v}{\partial z}\hat{a_z}$$

$$\nabla v = \frac{\partial v}{\partial r}\hat{a_r} + \frac{1}{r}\cdot\frac{\partial v}{\partial \theta}\hat{a_\theta} + \frac{1}{r\sin\theta}\cdot\frac{\partial v}{\partial \phi}\hat{a_\phi}$$

Ex:-

i) $v = 8x^2 y z \qquad \nabla v$

$$= 16xyz\,\hat{a_x} + 8x^2 z\,\hat{a_y} + 8x^2 y\,\hat{a_z}$$

ii) $v = 6\rho^2 (\sin\phi) z^2$

$$\nabla v = 12\rho(\sin\phi) z^2\,\hat{a_\rho} + \frac{1}{\cancel{\rho}}\, 6\rho^{\cancel{2}}(\cos\phi) z^2\,\hat{a_\rho}$$
$$+ 6\rho^2(\sin\phi)\, 2z\,\hat{a_z}$$

م.عبدالله/0501427704

iii) $V = 7r^2 \sin\theta \cos\phi$

$\nabla V = 14\, r \sin\theta \cos\phi \; a_r + 7r \cos\theta \cos\phi \; a_\theta$
$+ 7r(-\sin\phi)\; a_\phi$

5- divergence Theorm:- $\rightarrow$ scaler الناتج
مقدار

$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ cart

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$ cly

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$ sph

Ex :-

$$\vec{A} = \boxed{6xy^2z}\, \hat{a}_x + 8yz\, \hat{a}_y + \boxed{5x}\, \hat{a}_z$$

Ax $\quad$ 0

$$\nabla \cdot \vec{A} = 6y^2z + 8z + 0$$

$$\vec{A} = \boxed{3\rho \sin\phi}\, \hat{a}_\rho + \boxed{5\cos\phi}\, \hat{a}_\phi + \boxed{6z}\, \hat{a}_z$$

$A\rho$ $\quad$ $A\phi$ $\quad$ Az

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\emptyset}{\partial \emptyset} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\emptyset}{\partial \emptyset}$$

$$\nabla \cdot \vec{A} = \frac{1}{\boxed{\rho}} \frac{\partial}{\partial \rho}\left[3\rho^2 \sin\phi\right] + \frac{1}{\rho}\left[-5\sin\phi\right] + 6$$

$6\rho \sin\phi$

$$= 6\sin\phi - \frac{-5\sin\phi}{\rho} + 6$$

$$\vec{A} = 3r\sin\theta\, \hat{a}_r + 5\cos\theta \sin\phi\, \hat{a}_\phi + 6r\, \hat{a}_\phi$$

$$= 9\sin\theta + \frac{1}{r\sin\theta}\left[(5\sin\phi)(-\sin^2\theta + \cos^2\theta)\right] + 0$$

م.عبدالله/0501427704

الاوراق خاصة بالمشتركين لا احلل نشرها

6- Stoke's Theorem :- Vector ← الناتج طلع

$$\oint \vec{A}\cdot\vec{dl} = \iint \nabla\times\vec{A}\cdot ds$$

$$\nabla\times\vec{A} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \end{array}$$

$$\nabla\times\vec{A} = \frac{1}{\rho} \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{a}_\rho & \rho\,\hat{a}_\phi & \hat{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \end{array}$$

$$\nabla\times\vec{A} = \frac{1}{r^2\sin\theta} \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} a_r & r\,\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & r A_\theta & r\sin\theta\, A_\phi \end{vmatrix} \end{array}$$

Ex:-

$$\vec{A} = 6y\,\hat{a_x} + 5xy\,\hat{a_y} + 7z\,\hat{a_x}$$

$$\nabla \times \vec{A} = \begin{array}{ccc} + & - & + \\ \end{array}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6y & 5xy & 7z \end{vmatrix}$$

$$= 0\,\hat{a_x} - 0\,\hat{a_y} + [5y - 6]\,\hat{a_z}$$

$$= [5y - 6]\,\hat{a_z}$$

$$\vec{A} = 3\rho\sin\phi\,\hat{a_\rho} + 5z\,\hat{a_z}$$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a_\rho} & \rho\,\hat{a_\phi} & \hat{a_z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 3\rho\sin\phi & \rho\;0 & 5z \end{vmatrix}$$

$$= \boxed{\frac{1}{\rho}}\left[0\,\hat{a_\rho} - 0\,\hat{a_\phi} - 3\rho\cos\phi\,\hat{a_z}\right]$$

$$= -3\cos\phi\,\hat{a_z}$$

$$\vec{A} = 3r\sin\theta\, \hat{a}_r + 5\cos\theta\sin\phi\, \hat{a}_\theta + 9r\, \hat{a}_\phi$$

$$= \frac{1}{r^2\sin\theta}\Big[(9r^2\cos\theta - 3r\cos\theta\cos\phi)\hat{a}_r - (18r\sin\theta)r\,\hat{a}_\theta$$

$$+ (5\sin\phi\cos\theta - 3r\cos\theta\, r\sin\theta)\Big]$$