3- potential:- $V(r)$ Volt

$$\vec{E} = -\nabla V$$

$$V_{AB} = -\int_A^B E \cdot dL = V_B - V_A$$

special casese:-

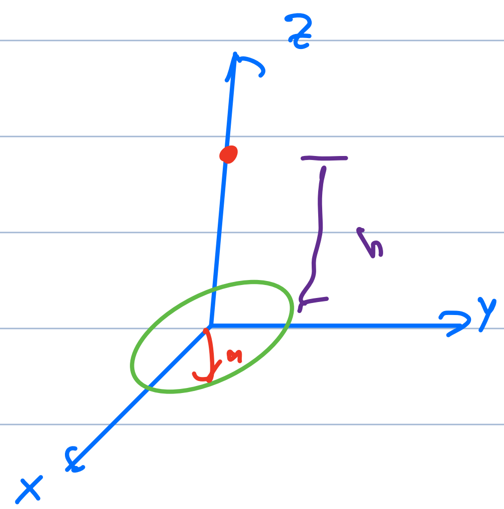
i) $$V_{AB} = V_B - V_A = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$ Point charge

ii) $$V_{AB} = V_B - V_A = \frac{-\rho_L}{2\pi\varepsilon_0} \ln \frac{\rho_B}{\rho_A}$$ line charge

iii) $$V_{AB} = V_B - V_A = \frac{-\rho_s}{2\varepsilon_0}\left[z_B - z_A\right]$$ sheet charge

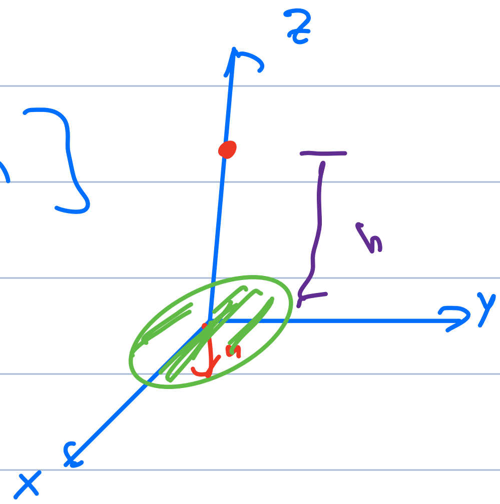
X potential for ring:-

$$V = \frac{\rho_L \, a}{2\varepsilon_0 \sqrt{a^2 + h^2}}$$



potential for disk :-

$$V = \frac{\rho_s}{2\varepsilon_0}\left[\sqrt{a^2+h^2} - h\right]$$



$$\rho_s = \frac{1}{\rho}$$

$\iint \rho_L$

infinite finite ring

2- Work:- energy to move

$$W = V\,Q$$
$$= -Q \int E\, dl$$

$$V = \frac{W}{Q}$$

2- energy stored in Electric field :- W Joule

$$W = \frac{1}{2} \int \vec{E} \cdot \vec{D}\, dv$$

$$\int\int\int$$

$$W = \frac{1}{2} \frac{1}{\varepsilon_0} \int \vec{D}^2\, dv$$

$$\boxed{W = \frac{1}{2} \varepsilon_0 \int \vec{E}^2\, dv}$$

3- energy density :- ω Joule $/m^3$

$$\omega = \frac{w}{V} = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{1}{\varepsilon}\vec{D}^2$$

$w \rightarrow$ energy stored

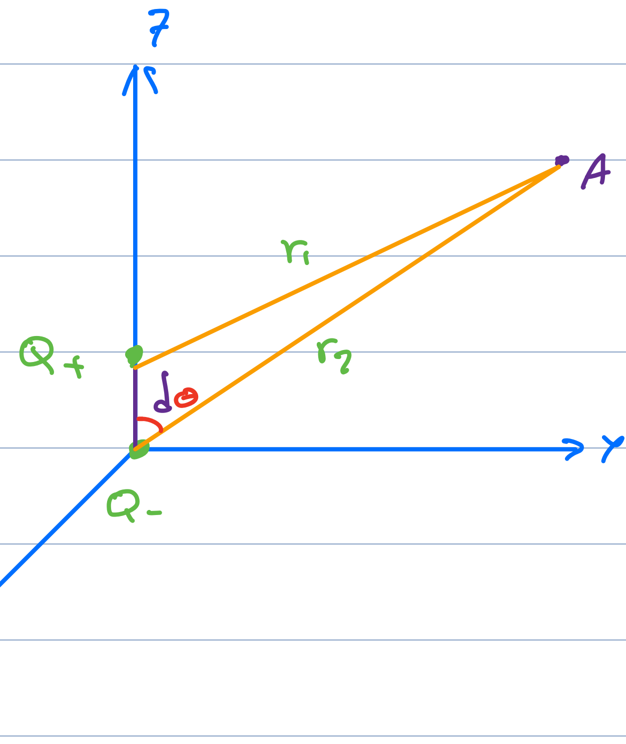
$V \rightarrow$ الحجم

4- dipole :-

$\vec{P} \rightarrow$ dipole moment

$$V = \frac{\boxed{\vec{P}}}{4\pi\varepsilon_0 |r - r'|^3} \cdot \langle r - r' \rangle$$

$$\vec{E} = -\nabla V$$



$P = \vec{P}\cos\theta$

$$\vec{E} = \frac{\boxed{P}}{4\pi\varepsilon_0 r^2}\left[2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta\right]$$

Tutorial - 4

Electrostatics

Chapter 4: Electrostatic Fields – Part II

Summary

7. The total work done, or the electric potential energy, to move a point charge Q from point A to B in an electric field $\mathbf{E}$ is

$$W = -Q\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

8. The potential at $\mathbf{r}$ due to a point charge Q at $\mathbf{r}'$ is

$$V(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_o|\mathbf{r}-\mathbf{r}'|} + C$$

where C is evaluated at a given reference potential point; for example, $C = 0$ if $V(\mathbf{r} \to \infty) = 0$. To determine the potential due to a continuous charge distribution, we replace Q in the formula for point charge by $dQ = \rho_L\, dl$, $dQ = \rho_S\, dS$, or $dQ = \rho_v\, dv$ and integrate over the line, surface, or volume, respectively.

9. If the charge distribution is not known, but the field intensity $\mathbf{E}$ is given, we find the potential by using

$$V = -\int_L \mathbf{E}\cdot d\mathbf{l} + C$$

10. The potential difference V_{AB}, the potential at B with reference to A, is

$$V_{AB} = -\int_A^B \mathbf{E}\cdot d\mathbf{l} = \frac{W}{Q} = V_B - V_A$$

11. Since an electrostatic field is conservative (the net work done along a closed path in a static $\mathbf{E}$ field is zero),

$$\oint_L \mathbf{E}\cdot d\mathbf{l} = 0$$

or

$$\nabla \times \mathbf{E} = \mathbf{0} \quad \text{(second Maxwell equation to be derived)}$$

12. Given the potential field, the corresponding electric field is found by using

$$\mathbf{E} = -\nabla V$$

13. For an electric dipole centered at $\mathbf{r}'$ with dipole moment $\mathbf{p}$, the potential at $\mathbf{r}$ is given by

$$V(\mathbf{r}) = \frac{\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r}-\mathbf{r}'|^3}$$

14. The flux density $\mathbf{D}$ is tangential to the electric flux lines at every point. An equipotential surface (or line) is one on which $V =$ constant. At every point, the equipotential line is orthogonal to the electric flux line.

15. The electrostatic energy due to n point charges is

$$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$$

For a continuous volume charge distribution,

$$W_E = \frac{1}{2}\int_v \mathbf{D}\cdot\mathbf{E}\, dv = \frac{1}{2}\int_v \varepsilon_o|\mathbf{E}|^2\, dv$$

16. Electrostatic discharge (ESD) refers to the sudden transfer of static charge between objects at different electrostatic potentials. Since all semiconductor devices are regarded as ESD sensitive, a good understandng of ESD is required in industry.

Textbook Practice Exercises

PRACTICE EXERCISE 4.10

If point charge 3 μC is located at the origin in addition to the two charges of Example 4.10, find the potential at $(-1, 5, 2)$, assuming $V(\infty) = 0$.

Answer: 10.23 kV.

$$V(\mathbf{r}) = \sum_{k=1}^{3} \frac{Q_k}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_k|} + C$$

$$At \quad V(\infty) = 0, \quad C = 0$$

$$|\mathbf{r} - \mathbf{r}_1| = |(-1,5,2) - (2,-1,3)| = \sqrt{46}$$

$$|\mathbf{r} - \mathbf{r}_2| = |(-1,5,2) - (0,4,-2)| = \sqrt{18}$$

$$|\mathbf{r} - \mathbf{r}_3| = |(-1,5,2) - (0,0,0)| = \sqrt{30}$$

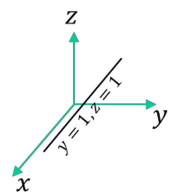
$$V(-1,5,2) = \frac{10^{-6}}{4\pi(\frac{10^{-9}}{36\pi})}\left[\frac{-4}{\sqrt{46}} + \frac{5}{\sqrt{18}} + \frac{3}{\sqrt{30}}\right]$$

$$= \underline{\underline{10.23\ kV}}$$

EXAMPLE 4.11

A point charge of 5 nC is located at $(-3, 4, 0)$, while line $y = 1$, $z = 1$ carries uniform charge 2 nC/m.

(a) If $V = 0$ V at $O(0, 0, 0)$, find V at $A(5, 0, 1)$.
(b) If $V = 100$ V at $B(1, 2, 1)$, find V at $C(-2, 5, 3)$.
(c) If $V = -5$ V at O, find V_{BC}.



Solution:

Let the potential at any point be

$$V = V_Q + V_L$$

where V_Q and V_L are the contributions to V at that point due to the point charge and the line charge, respectively. For the point charge,

$$V_Q = -\int \mathbf{E} \cdot d\mathbf{l} = -\int \frac{Q}{4\pi\varepsilon_o r^2} \mathbf{a}_r \cdot dr\, \mathbf{a}_r$$

$$= \frac{Q}{4\pi\varepsilon_o r} + C_1$$

For the infinite line charge,

$$V_L = -\int \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\rho_L}{2\pi\varepsilon_o \rho} \mathbf{a}_\rho \cdot d\rho\, \mathbf{a}_\rho$$

$$= -\frac{\rho_L}{2\pi\varepsilon_o} \ln \rho + C_2$$

Hence,

$$V = -\frac{\rho_L}{2\pi\varepsilon_o} \ln \rho + \frac{Q}{4\pi\varepsilon_o r} + C$$

where $C = C_1 + C_2 =$ constant, ρ is the perpendicular distance from the line $y = 1$, $z = 1$ to the field point, and r is the distance from the point charge to the field point.

(a) If $V = 0$ at $O(0, 0, 0)$, and V at $A(5, 0, 1)$ is to be determined, we must first determine the values of ρ and r at O and A. Finding r is easy; we use eq. (2.31). To find ρ for any point (x, y, z), we utilize the fact that ρ is the perpendicular distance from (x, y, z) to line $y = 1$, $z = 1$, which is parallel to the x-axis. Hence ρ is the distance between (x, y, z) and $(x, 1, 1)$ because the distance vector between the two points is perpendicular to $\mathbf{a}_x$. Thus

$$\rho = |(x, y, z) - (x, 1, 1)| = \sqrt{(y-1)^2 + (z-1)^2}$$

Applying this for ρ and eq. (2.31) for r at points O and A, we obtain

$$\rho_O = |(0, 0, 0) - (0, 1, 1)| = \sqrt{2}$$
$$r_O = |(0, 0, 0) - (-3, 4, 0)| = 5$$

$$\rho_A = |(5, 0, 1) - (5, 1, 1)| = 1$$
$$r_A = |(5, 0, 1) - (-3, 4, 0)| = 9$$

Hence,

$$V_O - V_A = -\frac{\rho_L}{2\pi\varepsilon_o} \ln \frac{\rho_O}{\rho_A} + \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_O} - \frac{1}{r_A}\right]$$

$$= \frac{-2 \cdot 10^{-9}}{2\pi \cdot \frac{10^{-9}}{36\pi}} \ln \frac{\sqrt{2}}{1} + \frac{5 \cdot 10^{-9}}{4\pi \cdot \frac{10^{-9}}{36\pi}}\left[\frac{1}{5} - \frac{1}{9}\right]$$

$$0 - V_A = -36 \ln \sqrt{2} + 45\left(\frac{1}{5} - \frac{1}{9}\right)$$

or

$$V_A = 36 \ln \sqrt{2} - 4 = 8.477 \text{ V}$$

Notice that we have avoided calculating the constant C by subtracting one potential from another and that it does not matter which one is subtracted from which.

(b) If $V = 100$ at $B(1, 2, 1)$ and V at $C(-2, 5, 3)$ is to be determined, we find

$$\rho_B = |(1, 2, 1) - (1, 1, 1)| = 1$$
$$r_B = |(1, 2, 1) - (-3, 4, 0)| = \sqrt{21}$$
$$\rho_C = |(-2, 5, 3) - (-2, 1, 1)| = \sqrt{20}$$
$$r_C = |(-2, 5, 3) - (-3, 4, 0)| = \sqrt{11}$$

$$V_C - V_B = -\frac{\rho_L}{2\pi\varepsilon_o} \ln \frac{\rho_C}{\rho_B} + \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_C} - \frac{1}{r_B}\right]$$

$$V_C - 100 = -36 \ln \frac{\sqrt{20}}{1} + 45 \cdot \left[\frac{1}{\sqrt{11}} - \frac{1}{\sqrt{21}}\right]$$

$$= -50.175 \text{ V}$$

or

$$V_C = 49.825 \text{ V}$$

(c) To find the potential difference between two points, we do not need a potential reference if a common reference is assumed.

$$V_{BC} = V_C - V_B = 49.825 - 100$$
$$= -50.175 \text{ V}$$

PRACTICE EXERCISE 4.11

A point charge of 5 nC is located at the origin. If $V = 2$ V at $(0, 6, -8)$, find

(a) The potential at $A(-3, 2, 6)$
(b) The potential at $B(1, 5, 7)$
(c) The potential difference V_{AB}

Answer: (a) 3.929 V, (b) 2.696 V, (c) -1.233 V.

$$V = \frac{Q}{4\pi\varepsilon_0 r} + C$$

If $V(0,6,-8) = V(r = 10) = 2$;

$$2 = \frac{5(10^{-9})}{4\pi(\frac{10^{-9}}{36\pi})(10)} + \boxed{C} \longrightarrow \boxed{C = -2.5}$$

(a)

$$V_A = \frac{5(10^{-9})}{4\pi(\frac{10^{-9}}{36\pi})|(-3,2,6) - (0,0,0)|} - 2.5$$

$$= \underline{\underline{3.929\ V}}$$

(b)

$$V_B = \frac{45}{\sqrt{7^2 + 1^2 + 5^2}} - 2.5 = \underline{\underline{2.696\ V}}$$

(c) $V_{AB} = V_B - V_A = 2.696 - 3.929 = \underline{\underline{-1.233\ V}}$

EXAMPLE 4.12

Given the potential $V = \frac{10}{r^2}\sin\theta\cos\phi$,

(a) Find the electric flux density **D** at $(2, \pi/2, 0)$.
(b) Calculate the work done in moving a 10 μC charge from point $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$.

Solution:
(a) $\mathbf{D} = \varepsilon_o\mathbf{E}$

But

$$\mathbf{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\mathbf{a}_\phi\right]$$
$$= \frac{20}{r^3}\sin\theta\cos\phi\,\mathbf{a}_r - \frac{10}{r^3}\cos\theta\cos\phi\,\mathbf{a}_\theta + \frac{10}{r^3}\sin\phi\,\mathbf{a}_\phi$$

At $(2, \pi/2, 0)$,

$$\mathbf{D} = \varepsilon_o\mathbf{E}\,(r = 2, \theta = \pi/2, \phi = 0) = \varepsilon_o\left(\frac{20}{8}\mathbf{a}_r - 0\mathbf{a}_\theta + 0\mathbf{a}_\phi\right)$$
$$= 2.5\varepsilon_o\mathbf{a}_r\ \text{C/m}^2 = 22.1\ \mathbf{a}_r\ \text{pC/m}^2$$

(b) The work done can be found in two ways, using either **E** or V.

Method 1:

$$W = -Q\int_L \mathbf{E}\cdot d\mathbf{l} \quad \text{or} \quad -\frac{W}{Q} = \int_L \mathbf{E}\cdot d\mathbf{l}$$

and because the electrostatic field is conservative, the path of integration is immaterial. Hence the work done in moving Q from $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$ is the same as that in moving Q from A to A', from A' to B', and from B' to B, where

$A(1, 30°, 120°)$		$B(4, 90°, 60°)$
$\downarrow d\mathbf{l} = dr\,\mathbf{a}_r$	$d\mathbf{l} = r\,d\theta\,\mathbf{a}_\theta$	$\uparrow d\mathbf{l} = r\sin\theta\,d\phi\,\mathbf{a}_\phi$
$A'(4, 30°, 120°)$	$\rightarrow$	$B'(4, 90°, 120°)$

That is, instead of being moved directly from A to B, Q is moved from $A \to A'$, $A' \to B'$, $B' \to B$, so that only one variable is changed at a time. This makes the line integral much easier to evaluate. Thus

$$\frac{-W}{Q} = -\frac{1}{Q}(W_{AA'} + W_{A'B'} + W_{B'B})$$
$$= \left(\int_{AA'} + \int_{A'B'} + \int_{B'B}\right)\mathbf{E}\cdot d\mathbf{l}$$
$$= \int_{r=1}^{4}\frac{20\sin\theta\cos\phi}{r^3}dr\Big|_{\theta=30°,\phi=120°}$$
$$+ \int_{\theta=30°}^{90°}\frac{-10\cos\theta\cos\phi}{r^3}r\,d\theta\Big|_{r=4,\phi=120°}$$
$$+ \int_{\phi=120°}^{60°}\frac{10\sin\phi}{r^3}r\sin\theta\,d\phi\Big|_{r=4,\theta=90°}$$
$$= 20\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left[-\frac{1}{2r^2}\Big|_{r=1}^{4}\right]$$
$$- \frac{10}{16}\frac{(-1)}{2}\sin\theta\Big|_{30°}^{90°} + \frac{10}{16}(1)\left[-\cos\phi\Big|_{120°}^{60°}\right]$$
$$-\frac{W}{Q} = \frac{-75}{32} + \frac{5}{32} - \frac{10}{16}$$

or

$$W = \frac{45}{16}Q = 28.125\ \mu\text{J}$$

Method 2:
Since V is known, this method is much easier.

$$W = -Q\int_A^B \mathbf{E}\cdot d\mathbf{l} = QV_{AB}$$
$$= Q(V_B - V_A)$$
$$= 10\left(\frac{10}{16}\sin 90°\cos 60° - \frac{10}{1}\sin 30°\cos 120°\right)\cdot 10^{-6}$$
$$= 10\left(\frac{10}{32} - \frac{-5}{2}\right)\cdot 10^{-6}$$
$$= 28.125\ \mu\text{J as obtained before}$$

PRACTICE EXERCISE 4.12

Given that $\mathbf{E} = (3x^2 + y)\mathbf{a}_x + x\mathbf{a}_y$ kV/m, find the work done in moving a $-2\ \mu$C charge from $(0, 5, 0)$ to $(2, -1, 0)$ by taking the straight-line path.

(a) $(0, 5, 0) \to (2, 5, 0) \to (2, -1, 0)$
(b) $y = 5 - 3x$

Answer: (a) 12 mJ, (b) 12 mJ.

(a)

$$\frac{-W}{Q} = \int \boldsymbol{E}\bullet d\boldsymbol{l} = \int (3x^2+y)dx + x dy$$
$$= \int_0^2 (3x^2+y)dx\Big|_{y=5} + \int_5^{-1} x\,dy\Big|_{x=2}$$
$$= 18 - 12 = 6\ \text{kV}$$
$$W = -6Q = \underline{\underline{12\ \text{mJ}}}$$

(b)

$$dy = -3dx$$
$$-\frac{W}{Q} = \int \boldsymbol{E}\bullet d\boldsymbol{l} = \int_0^2 (3x^2+5-3x)dx + x(-3)dx$$
$$= \int_0^2 (3x^2-6x+5)dx = 8 - 12 + 10 = 6$$
$$W = \underline{\underline{12\ \text{mJ}}}$$

PRACTICE EXERCISE 4.13

An electric dipole of 100 $\mathbf{a}_z$ pC · m is located at the origin. Find V and **E** at points

(a) $(0, 0, 10)$
(b) $(1, \pi/3, \pi/2)$

Answer: (a) 9 mV, 1.8$\mathbf{a}_r$ mV/m, (b) 0.45 V, $0.9\mathbf{a}_r + 0.7794\mathbf{a}_\theta$ V/m.

(a) $(0,0,10) \longrightarrow (r=10, \theta=0, \phi=0)$

$$V = \frac{100\cos 0}{4\pi\varepsilon_0(10^2)}(10^{-12}) = \frac{10^{-12}}{4\pi(\frac{10^{-9}}{36\pi})} = \underline{\underline{9\ \text{mV}}}$$

$$E = \frac{100(10^{-12})}{4\pi(\frac{10^{-9}}{36\pi})10^3}[2\cos 0\,\boldsymbol{a}_r + \sin 0\,\boldsymbol{a}_\theta]$$
$$= \underline{\underline{1.8\ \boldsymbol{a}_r\ \text{mV/m}}}$$

(b) At $(1, \frac{\pi}{3}, \frac{\pi}{2})$,

$$V = \frac{100\cos\frac{\pi}{3}(10^{-12})}{4\pi(\frac{10^{-9}}{36\pi})(1)^2} = \underline{\underline{0.45\ V}}$$

$$E = \frac{100(10^{-12})}{4\pi(\frac{10^{-9}}{36\pi})(1)^2}(2\cos\frac{\pi}{3}\boldsymbol{a}_r + \sin\frac{\pi}{3}\boldsymbol{a}_\theta)$$
$$= \underline{\underline{0.9\,\boldsymbol{a}_r + 0.7794\boldsymbol{a}_\theta\ \ \text{V/m}}}$$

$V_{21} =$

PRACTICE EXERCISE 4.14

Point charges $Q_1 = 1$ nC, $Q_2 = -2$ nC, $Q_3 = 3$ nC, and $Q_4 = -4$ nC are positioned one at a time and in that order at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 0, -1)$, and $(0, 0, 1)$, respectively. Calculate the energy in the system after each charge is positioned.

Answer: 0, -18 nJ, -29.18 nJ, -68.27 nJ.

After Q_1, $W_1 = 0$

After Q_2, $W_2 = \boxed{Q_2 V_{21}} = \dfrac{Q_2\, Q_1}{4\pi\varepsilon_0 |(1,0,0)-(0,0,0)|}$

$$= \frac{1(-2)(10^{-18})}{4\pi(10^{-9})\dfrac{1}{36\pi}} = \underline{-18 \text{ nJ}}$$

After Q_3,

$$W_3 = Q_3(V_{31}+V_{32}) + Q_2V_{21}$$

$$= 3(9)(10^{-9})\left\{\frac{1}{|(0,0,-1)-(0,0,0)|} + \frac{-2}{|(0,0,-1)-(1,0,0)|}\right\} - 18 \text{ nJ}$$

$$= 27\left(1-\frac{2}{\sqrt{2}}\right) - 18$$

$$= \underline{\underline{-29.18 \text{ nJ}}}$$

After Q_4,

$$W_4 = Q_4(V_{41}+V_{42}+V_{43}) + Q_3(V_{31}+V_{32}) + Q_2V_{21}$$

$$= -4(9)(10^{-9})\left\{\frac{1}{|(0,0,1)-(0,0,0)|} + \frac{-2}{|(0,0,1)-(1,0,0)|} + \frac{3}{|(0,0,1)-(0,0,-1)|}\right\} + W_3$$

$$= -36\left(1-\frac{2}{\sqrt{2}}+\frac{3}{2}\right) + W_3$$

$$= -39.09 - 29.18 \text{ nJ} = \underline{\underline{-68.27 \text{ nJ}}}$$

EXAMPLE 4.15

A charge distribution with spherical symmetry has density

$$\rho_v = \begin{cases} \rho_o, & 0 \le r \le R \\ 0, & r > R \end{cases}$$

Determine V everywhere and the energy stored in region $r < R$.

Solution:

The **D** field has already been found in Section 4.6D using Gauss's law.

(a) For $r \ge R$, $\mathbf{E} = \dfrac{\rho_o R^3}{3\varepsilon_o r^2}\mathbf{a}_r$.

Once **E** is known, V is determined as

$$V = -\int \mathbf{E}\cdot d\mathbf{l} = -\frac{\rho_o R^3}{3\varepsilon_o}\int \frac{1}{r^2}dr$$

$$= \frac{\rho_o R^3}{3\varepsilon_o r} + C_1, \quad r \ge R$$

Since $V(r=\infty) = 0$, $C_1 = 0$.

(b) For $r \le R$, $\mathbf{E} = \dfrac{\rho_o r}{3\varepsilon_o}\mathbf{a}_r$.

Hence,

$$V = -\int \mathbf{E}\cdot d\mathbf{l} = -\frac{\rho_o}{3\varepsilon_o}\int r\, dr$$

$$= -\frac{\rho_o r^2}{6\varepsilon_o} + C_2$$

From part (a) $V(r=R) = \dfrac{\rho_o R^2}{3\varepsilon_o}$. Hence,

$$\frac{R^2\rho_o}{3\varepsilon_o} = -\frac{R^2\rho_o}{6\varepsilon_o} + C_2 \rightarrow C_2 = \frac{R^2\rho_o}{2\varepsilon_o}$$

and

$$V = \frac{\rho_o}{6\varepsilon_o}(3R^2 - r^2)$$

Thus from parts (a) and (b)

$$V = \begin{cases} \dfrac{\rho_o R^3}{3\varepsilon_o r}, & r \ge R \\ \dfrac{\rho_o}{6\varepsilon_o}(3R^2 - r^2), & r \le R \end{cases}$$

(c) The energy stored is given by

$$W = \frac{1}{2}\int_v \mathbf{D}\cdot\mathbf{E}\, dv = \frac{1}{2}\varepsilon_o\int_v E^2\, dv$$

For $r \le R$,

$$\mathbf{E} = \frac{\rho_o r}{3\varepsilon_o}\mathbf{a}_r$$

Hence,

$$W = \frac{1}{2}\varepsilon_o\frac{\rho_o^2}{9\varepsilon_o^2}\int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2\cdot r^2\sin\theta\, d\phi\, d\theta\, dr$$

$$= \frac{\rho_o^2}{18\varepsilon_o}4\pi\cdot\frac{r^5}{5}\bigg|_0^R = \frac{2\pi\rho_o^2 R^5}{45\varepsilon_o}\text{J}$$

PRACTICE EXERCISE 4.15

If $V = x - y + xy + 2z$ V, find **E** at $(1, 2, 3)$ and the electrostatic energy stored in a cube of side 2 m centered at the origin.

Answer: $-3\mathbf{a}_x - 2\mathbf{a}_z$ V/m, 0.2358 nJ.

$$\mathbf{E} = -\nabla V = -(y+1)\mathbf{a}_x + (1-x)\mathbf{a}_y - 2\mathbf{a}_z$$

At (1,2,3), $\mathbf{E} = \underline{\underline{-3\mathbf{a}_x - 2\mathbf{a}_z \text{ V/m}}}$

$$W = \frac{1}{2}\varepsilon_o\int \mathbf{E}\bullet\mathbf{E}dv = \frac{1}{2}\varepsilon_o\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}(x^2+y^2-2x+2y+6)dxdydz = \frac{1}{2}\varepsilon_o\left[\int_{-1}^{1}x^2dx\iint dydz + \int_{-1}^{1}y^2dy\iint dxdz - 2\int_{-1}^{1}xdx\iint dydz + 2\int_{-1}^{1}ydy\iint dxdz + 6(2)(2)(2)\right]$$

$$= \frac{1}{2}\varepsilon_o\left[2\frac{x^3}{3}\bigg|_{-1}^{1}(2)(2) + 0 + 0 + 6(8)\right] = \frac{80\varepsilon_o}{3} \quad = \underline{\underline{0.2358 \text{ nJ}}}$$

----- End of Tutorial Document -----

PRACTICE EXERCISE 4.10

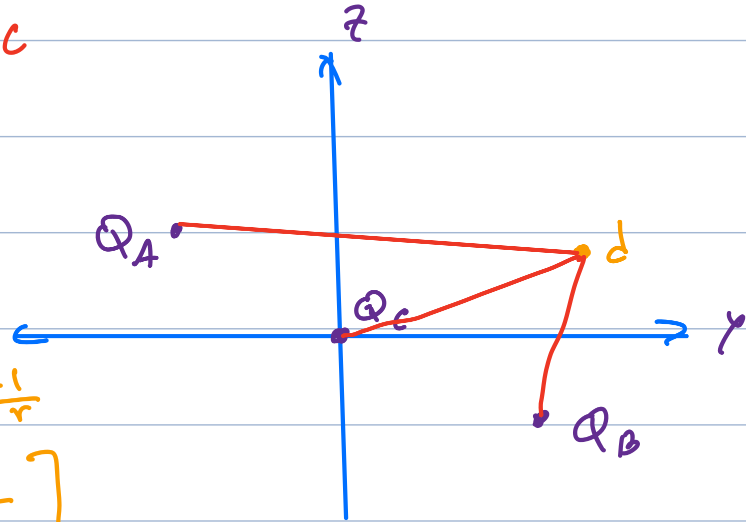
If point charge 3 μC is located at the origin in addition to the two charges of Example 4.10, find the potential at $(-1, 5, 2)$, assuming $V(\infty) = 0$.

Answer: 10.23 kV.

$Q = -4\ \mu C \quad (2, -1, 3)\ A \qquad (-1, 5, 2)\ d$

$Q = 5\ \mu C \quad (0, 4, -2)\ B$

$Q = 3\ \mu C \quad (0, 0, 0)\ C$



$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

$V = -\int \dfrac{Q}{4\pi\varepsilon_0 r^2} dr$

$= -\dfrac{Q}{4\pi\varepsilon_0} \boxed{\int \dfrac{1}{r^2}} \rightarrow -\dfrac{1}{r}$

$V_{AB} = V_B - V_A = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{r_B} - \dfrac{1}{r_A}\right]$

$V = \dfrac{Q}{4\pi\varepsilon_0 r}$

$V = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{Q_A}{r_A} + \dfrac{Q_B}{r_B} + \dfrac{Q_C}{r_C}\right]$

$r_A = (-1, 5, 2) - (2, -1, 3) = (-3, 6, -1)$

$|r_A| = \sqrt{46}$

$|r_B| = \sqrt{18}$

$|r_C| = \sqrt{30}$

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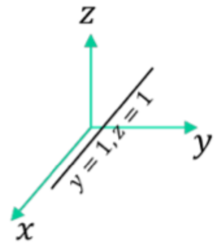
$= \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{-4\mu}{\sqrt{46}} + \dfrac{5\mu}{\sqrt{18}} + \dfrac{3}{\sqrt{30}}\right]$

$= 10.23$ kV

EXAMPLE 4.11

A point charge of 5 nC is located at $(-3, 4, 0)$, while line $y = 1$, $z = 1$ carries uniform charge 2 nC/m.

(a) If $V = 0$ V at $O(0, 0, 0)$, find V at $A(5, 0, 1)$.
(b) If $V = 100$ V at $B(1, 2, 1)$, find V at $C(-2, 5, 3)$.
(c) If $V = -5$ V at O, find V_{BC}.



Solution:

Let the potential at any point be

$$V = V_Q + V_L$$

i) $V_{AB} = V_B - V_A = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$ charge

ii) $V_{AB} = V_B - V_A = \frac{-\rho_L}{2\pi\varepsilon_0} \ln \frac{\rho_B}{\rho_A}$ line charge

a)

$$\boxed{V_{OA\,point}} = \boxed{V_A} \boxed{-V_O} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_o}\right]$$

?

0

$$V_{OA\,Line} = V_A - V_O = \frac{-\rho_L}{2\pi\varepsilon_0} \ln \frac{\rho_A}{\rho_o}$$

$r_A = (5, 0, 1) - (-3, 4, 0)$

$|r_A| = 9$

$r_o = (0, 0, 0) - (-3, 4, 0)$

$|r_o| = 5$

$$V_{OA\,point} = \frac{5\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{9} - \frac{1}{5}\right]$$

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PRACTICE EXERCISE 4.11

A point charge of 5 nC is located at the origin. If $V = 2$ V at $(0, 6, -8)$, find

(a) The potential at $A(-3, 2, 6)$
(b) The potential at $B(1, 5, 7)$ V_{oB}
(c) The potential difference V_{AB}

Answer: (a) $\boxed{3.929}$ V, (b) 2.696 V, (c) −1.233 V.

$$V = -\int E \, \boxed{dl}$$

$$= -\int \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= -\frac{Q}{4\pi\varepsilon_0} \int \frac{1}{r^2} dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right] = \frac{Q}{4\pi\varepsilon_0 r} \boxed{+C}$$

$$V_{oA} = V_A \boxed{-V_o} = \boxed{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_o}\right]}$$

$$V_A = V_{oA} + V_o$$

$$V_{oA} = \frac{5 \times 10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{7} - \frac{1}{10}\right]$$

$$r_o = (0, 6, -8) - (0, 0, 0)$$

$$r_o = (0, 6, -8)$$

$$|r_o| = 10$$

$(0, 6, -8)$ $r = 10$

$$2 = \frac{5 \times 10^{-9}}{4\pi\varepsilon_0 (10)} + C$$

$$C = -2,5$$

a)

$$V_A = \frac{5}{4\pi\varepsilon_0 (7)} - 2,5 =$$

b)

$$V_B =$$

$$r_A = (-3, 2, 6) - (0, 0, 0)$$

$$r_A = (-3, 2, 6)$$

$$|r_A| = 7$$

c) $V_{AB} = V_B - V_A = 2,696 - 3,929 = -1,233$

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EXAMPLE 4.12

Given the potential $V = \frac{10}{r^2} \sin\theta \cos\phi$, → sphere

(a) Find the electric flux density **D** at $(2, \pi/2, 0)$.
(b) Calculate the work done in moving a 10 μC charge from point $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$.

a) $E = -\nabla V =$

$$\mathbf{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

$$= \boxed{-}\Big[\frac{-20}{r^3}\sin\theta\cos\phi\,\hat{a_r} + \frac{1}{r}\cdot\frac{10}{r^2}\cos\theta\cos\phi\,\hat{a_\theta} - \frac{1}{r\sin\theta}\cdot\frac{10}{r^2}\sin\theta\sin\phi\,\hat{a_\phi}$$

$$\vec{E} = \frac{20}{r^3}\sin\theta\cos\phi\,\boxed{\hat{a_r}} - \frac{10}{r^3}\cos\theta\cos\phi\,\boxed{\hat{a_\theta}} + \frac{10}{r^3}\sin\phi\,\boxed{\hat{a_\phi}}$$

$\vec{D}$ @ $(r = 2,\ \theta = \pi/2,\ \phi = 0)$

$$\vec{D} = \varepsilon_0\vec{E}$$

$$\vec{D} = \varepsilon_0\Big[\frac{20}{8}\hat{a_r} - 0\,\hat{a_\theta} - 0\,\hat{a_\phi}\Big]$$

b) $W = -Q\int E\,dL$

$A(1, 30°, 120°)$

$dL = dr\,\hat{a_r}$

$(4, 30°, 120°)$ → $(4, 90°, 120°)$ → $B(4, 90°, 60°)$

$dL = r\,d\theta\,\hat{a_\theta}$

$dL = r\sin\theta\,d\phi\,\hat{a_\phi}$

$$W = -Q\left[\int_{1}^{4} \frac{20}{\underset{\downarrow}{r^3}} \underset{\substack{\downarrow \\ 30^\circ}}{\sin\theta}\ \underset{\substack{\downarrow \\ 120^\circ}}{\cos\phi}\, dr + \int_{30^\circ}^{90^\circ} \frac{-10}{\underset{\substack{\downarrow \\ 4}}{r^3}} \cos\theta\ \underset{\substack{\downarrow \\ 120^\circ}}{\cos\phi}\ r\, d\theta\right.$$

$$\boxed{+\boxed{\int_{120}^{60^\circ}} \frac{10\sin\phi}{\underset{\substack{\downarrow \\ 4}}{r^3}}\ r\ \underset{\substack{\downarrow \\ 90^\circ}}{\sin\theta}\ d\phi}$$

PRACTICE EXERCISE 4.12

Given that $\mathbf{E} = (3x^2 + y)\mathbf{a}_x + x\mathbf{a}_y$ kV/m, find the work done in moving a $-2\ \mu$C charge from (0, 5, 0) to (2, −1, 0) by taking the straight-line path.

(a) (0, 5, 0) → (2, 5, 0) → $\boxed{(2, -1, 0)}$
(b) $y = 5 - 3x$

Answer: (a) 12 mJ, (b) 12 mJ.

a)

(0, 5, 0) → (2, 5, 0) → (2, −1, 0)

$dl = dx$

$dl = dy$

$$W = -Q \int E\, dl$$

$$= 2\times10^{-6}\left[\int_0^2 3x^2 + y\, dx + \int_5^{-1} x\, dy\right]$$

$$W = 12\ mJ$$

b)

$$W = -Q \int E\, dl$$

$$= 2\times10^{-6}\left[\int_0^2 3x^2 + (5-3x)\, dx + \int_0^2 x\,(-3\, dx)\right]$$

$y = \boxed{5 - 3x}$

$dy = \boxed{-3\, dx}$

CH4

Table - 5: Electrostatics Relations

Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12} \approx 10^{-9}/36\pi$ F/m. Permittivity of Dielectric: $\varepsilon = \varepsilon_0 \varepsilon_r$ F/m.

$\mathbf{F} = \boxed{\frac{Q_1 Q_2}{4\pi\varepsilon R^2}} \mathbf{a}_R$	Force (N) (point charge)	$\mathbf{F} = \frac{Q}{4\pi\varepsilon}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{\lvert\mathbf{r}-\mathbf{r}_k\rvert^3}$	Force (N) (superposition)
$\mathbf{E} = \frac{\mathbf{F}}{Q} = -\nabla V = \frac{Q}{4\pi\varepsilon R^2}\mathbf{a}_R$	Electric field intensity (point charge) (V/m) (N/C)	$\mathbf{E} = \frac{1}{4\pi\varepsilon}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{\lvert\mathbf{r}-\mathbf{r}_k\rvert^3}$	Electric field intensity (superposition) (V/m) (N/C)
$\mathbf{D} = \varepsilon\, \mathbf{E}$	Electric flux density (C/m²)	$\psi = \int_S \mathbf{D}\cdot d\mathbf{S}$	Electric flux (C)
$\psi = Q_{enc} = \oint_S \mathbf{D}\cdot d\mathbf{S}$ (integral) $\rho_v = \nabla\cdot\mathbf{D}$ (differential)	Gauss's law (first Maxwell's equation)	$W = V_L = -Q\int_A^B \mathbf{E}\cdot d\mathbf{l} = -\int_L \mathbf{E}\cdot d\mathbf{l} + C$	Total work (J) (potential energy)
$V = \frac{Q}{4\pi\varepsilon\, r} = \frac{Q}{4\pi\varepsilon\,\lvert\mathbf{r}-\mathbf{r}`\rvert} + C$	Potential at $\mathbf{r}$ due to point charge Q (centered at $\mathbf{r}`$)	$V_{AB} = V_B - V_A = \frac{W}{Q} = -\int_L \mathbf{E}\cdot d\mathbf{l}$	Potential difference (J/C =V)
$V(\mathbf{r}) = \frac{\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}`)}{4\pi\varepsilon\,\lvert\mathbf{r}-\mathbf{r}`\rvert^3} + C$	Potential at $\mathbf{r}$ due to dipole with moment $\mathbf{p}$ (centered at $\mathbf{r}`$)	$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$	Electrostatic energy (J) due to n point charges
$V = \frac{\mathbf{p}\cdot\boldsymbol{a}_r}{4\pi\varepsilon\, r^2}$	Potential at $\mathbf{r}$ due to dipole with moment $\mathbf{p}$ (centered at origin)	$\mathbf{E} = \frac{p}{4\pi\varepsilon\, r^3}(2\cos\theta\,\boldsymbol{a}_r + \sin\theta\,\boldsymbol{a}_\theta)$	Intensity due to dipole at origin: $p = \lvert\mathbf{p}\rvert = Qd$
$W = \oint_L \mathbf{E}\cdot d\mathbf{l} = 0$ (integral) $\nabla\times\mathbf{E} = 0$ (differential)	Kirchhoff's law (second Maxwell's equation)	$W_E = \frac{1}{2}\int_v \mathbf{D}\cdot\mathbf{E}\, dv = \frac{1}{2}\int_v \varepsilon\,\lvert\mathbf{E}\rvert^2\, dv$	Electrostatic energy (J) (continuous volume charge)

CH5

$\mathbf{J} = \frac{\Delta I}{\Delta S} = \rho_v\,\mathbf{u} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\,\mathbf{E}$	Current density (A/m²) σ: conductivity (℧/m) (S/m)	$I = \frac{dQ}{dt} = \int_S \mathbf{J}\cdot d\mathbf{S}$	Current (A)
$R = \frac{V}{I} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \mathbf{J}\cdot d\mathbf{S}} = \frac{\rho_c\, l}{S}$	Resistance (uniform) Ω $\rho_c = 1/\sigma$: resistivity	$P = V\,I = I^2 R = \int_v \mathbf{E}\cdot\mathbf{J}\, dv$	Power (uniform) W
$\rho_{ps} = \mathbf{P}\cdot\boldsymbol{a}_n$ $\rho_{pv} = -\nabla\cdot\mathbf{P}$	Bound (or polarization) surface and volume charge densities	$\mathbf{P} = \chi_e\varepsilon_0\mathbf{E}$	Polarization intensity (C/m²) $\chi_e = \varepsilon_r - 1$: susceptibility
$\nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}$	Continuity equation	$T_r = \frac{\varepsilon}{\sigma}$	Relaxation time (sec)
$E_{1t} = E_{2t},\ D_{1n} - D_{2n} = \rho_s$ or $D_{1n} = D_{2n}$ (if $\rho_s = 0$)	Boundary conditions (Dielectric- Dielectric)	$D_t = \varepsilon E_t = 0,\quad D_n = \varepsilon E_n = \rho_s$	Boundary conditions (Conductor- Dielectric)
$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$	Law of refraction	$D_t = \varepsilon_0 E_t = 0,\quad D_n = \varepsilon_0 E_n = \rho_s$	Boundary conditions (Conductor- Free)

CH6

$\nabla\cdot\varepsilon\nabla V = -\rho_v$ (inhomogeneous) $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ (homogeneous)	Poisson's equation	$\nabla^2 V = 0$	Laplace's equation
$C = \frac{Q}{V} = \frac{\int_S \mathbf{D}\cdot d\mathbf{S}}{\int_L \mathbf{E}\cdot d\mathbf{l}}$	Capacitance (F)	$RC = \frac{\varepsilon}{\sigma}$	RC relation
$C = \frac{\varepsilon\, S}{d}$	Parallel-Plate capacitance (F)	$C = \frac{2\pi\varepsilon\, L}{\ln\frac{b}{a}}$	Coaxial capacitance (F)
$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$	Spherical capacitance (F)	$4\pi\varepsilon\, a$	Isolated sphere capacitance (F)

	Line	Surface	Volume	Unit
Total charge $Q =$	$\int_L \rho_L\, dl$	$\int_S \rho_S\, dS$	$\int_v \rho_v\, dv = Q_{enc}$	C
Field intensity $\mathbf{E} =$ (finite charge distribution)	$\int_L \frac{\rho_L\, dl}{4\pi\varepsilon\, R^2}\mathbf{a}_R$	$\int_S \frac{\rho_S\, dS}{4\pi\varepsilon\, R^2}\mathbf{a}_R$	$\int_v \frac{\rho_v\, dv}{4\pi\varepsilon\, R^2}\mathbf{a}_R$	N/C or (V/m)
Field intensity $\mathbf{E} =$ (infinite charge distribution)	$\frac{\rho_L}{2\pi\varepsilon\,\rho}\mathbf{a}_\rho$ ρ : distance to field point	$\frac{\rho_S}{2\varepsilon}\mathbf{a}_n$	$\frac{Q}{4\pi\varepsilon\, r^2}\mathbf{a}_r$ same as of a **point** charge	
Field density $\mathbf{D} =$ (using Gauss's law)	$\frac{\rho_L}{2\pi\rho}\mathbf{a}_\rho$	$\frac{\rho_S}{2}\mathbf{a}_n$	$\begin{cases} \frac{r}{3}\rho_o\mathbf{a}_r, & 0 < r \le a \\ \frac{a^3}{3r^2}\rho_o\mathbf{a}_r, & r \ge a \end{cases}$	C/m (line) C/m² (surface) C/m³ (volume)

$W =$
$W =$
$\vec{D} = \epsilon_0 \vec{E}$
$\vec{F} = Q \vec{E}$
$V = - \int \vec{E} \, dL$

CH 4

E , D , V , F

point Q c

$E = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r}$

line ρ_L c/m

1- finite line
2- infinite line $E = \frac{\rho_L}{2\pi\epsilon_0} \hat{a_p}$
3- ring

ρ_s c/m²
Sheet
1- $E = \frac{\rho_s}{2\epsilon_0} \hat{a_r}$
2- desk

dipole
$\vec{E} = \text{-- -- --}$

ρ_v c/m³
Volume
E Gauss-law

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PRACTICE EXERCISE 4.13

An electric dipole of 100 $\mathbf{a}_z$ pC · m is located at the origin. Find V and $\mathbf{E}$ at points

(a) (0, 0, 10) $\rightarrow r = 10$, $\theta = 0$, $\phi = 0$

(b) $(1, \pi/3, \pi/2)$ $\rightarrow \phi$, θ

Answer: (a) 9 mV, $\boxed{1.8\mathbf{a}_r}$ mV/m, (b) 0.45 V, $0.9\mathbf{a}_r + 0.7794\mathbf{a}_\theta$ V/m.

a)

$$V = \frac{\boxed{P \; \hat{a}_r}}{4\pi \varepsilon_0 r^2} \rightarrow \vec{P} \cos\theta \qquad E = \frac{\boxed{\vec{P}}}{4\pi \varepsilon_0 r^3}\left[2\cos\theta \, \hat{a}_r + \sin\theta \, \hat{a}_\theta\right]$$

$$V = \frac{\vec{P}\cos\theta}{4\pi \varepsilon_0 r^2} \qquad = \boxed{\frac{100 \times 10^{-12}}{4\pi \varepsilon_0 (10)^3}}\left[2\cos 0 \, \hat{a}_r + \sin 0 \, \hat{a}_\theta\right]$$

$$= 1.8 \, \hat{a}_r \text{ m V/m}$$

$$= \frac{100 \times 10^{-12} \cos 0}{4\pi \varepsilon_0 (10)^2} = 9 \text{ mV}$$

b)

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PRACTICE EXERCISE 4.14

Point charges $Q_1 = 1$ nC, $Q_2 = -2$ nC, $Q_3 = 3$ nC, and $Q_4 = -4$ nC are positioned one at a time and in that order at (0, 0, 0), (1, 0, 0), (0, 0, −1), and (0, 0, 1), respectively. Calculate the energy in the system after each charge is positioned.

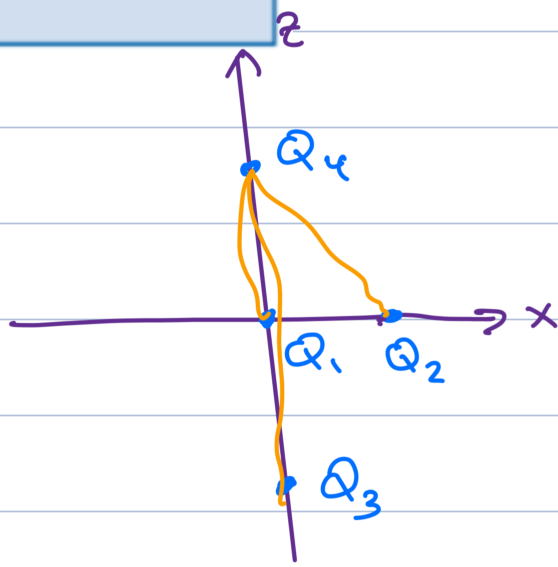
Answer: 0, −18 nJ, −29.18 nJ, −68.27 nJ.

$$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$$

$w_1 = 0$

$\boxed{w = Q\,V}$

$w = -Q \int E\,dl$



$w_2 = Q_2 V_{21}$

$w_3 = w_1 + w_2 + Q_3 (V_{31} + V_{32})$

$= Q_2 V_{21} + Q_3 (V_{31} + V_{32})$

$w_4 = w_1 + w_2 + w_3 + Q_4 (V_{41} + V_{42} + V_{43})$

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PRACTICE EXERCISE 4.15

If $V = x - y + xy + 2z$ V, find **E** at (1, 2, 3) and the electrostatic energy stored in a cube of side 2 m centered at the origin.

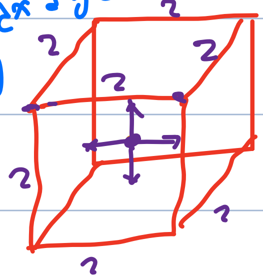
Answer: $-3\mathbf{a}_x - 2\mathbf{a}_z$ V/m, 0.2358 nJ.

$$E = -\nabla V = -\boxed{\left[(1+y)\,a_x + (-1+x)\,a_y + 2\,a_z\right]}$$

$$E = \boxed{-3\,a_x - 2\,a_z}$$

$$w = \frac{1}{2}\varepsilon_0 \iiint \vec{E}^2 \, dv \qquad dv \to dx\,dy\,dz$$

$$w = \frac{1}{2}\varepsilon_0 \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} (x^2+y^2-2x+2y+6) \quad dx\,dy\,dz$$



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EXAMPLE 4.15

A charge distribution with spherical symmetry has density

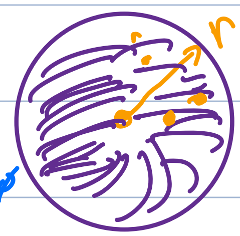
$$\rho_v = \begin{cases} \boxed{\rho_o}, & 0 \le r \le R \\ 0, & r > R \end{cases}$$

Determine V everywhere and the energy stored in region $r < R$.

$r \le R$

$$\oint \vec{D} \cdot d\vec{s} = Q_{net}$$

$$\vec{D}\, 4\pi r^2 = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho_o\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$



$$\vec{D}\, \cancel{4\pi} r^2 = \frac{\cancel{4}}{3} \cancel{\pi} R^3 \rho_o$$

$$\vec{D} = \frac{\rho_o R^3}{3 r^2}\, \hat{a}_r$$

$$\vec{E} = \frac{\rho_o R^3}{3 \varepsilon_o r^2}\, \hat{a}_r$$

$$V = -\int E\, dl = -\int \boxed{\frac{\rho_o R^3}{3\varepsilon_o r^2}}\, dr$$

$$= \frac{-\rho_o R^3}{3\varepsilon_o} \boxed{\int \frac{1}{r^2}\, dr} \rightarrow \frac{-1}{r}$$

$$V = \frac{\rho_o R^3}{3\varepsilon_o r} + c_1$$

$r > R$

$$\vec{D}\, 4\pi r^2 = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_o\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\vec{D}\, \cancel{4\pi} \cancel{r^2} = \frac{\cancel{4}}{3} \cancel{\pi}\, r^{\cancel{3}} \rho_o$$

$$\vec{D} = \frac{\rho_0 r}{3} a_r$$

$$\vec{E} = \frac{\rho_0 r}{3 \varepsilon_0} a_r$$

$$V = -\int E \, dl = -\frac{\rho_0}{3 \varepsilon_0} \int r \, dr$$

$$= -\frac{\rho_0 r^2}{6 \varepsilon_0} + C_2$$