Chapter 4:

Sect. 4.3: Q14:

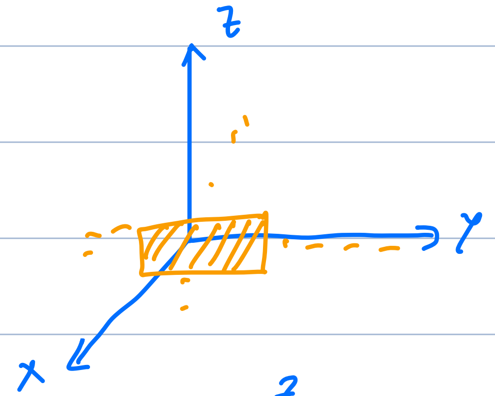
4.14 (a) An infinite sheet at $z = 0$ has a uniform charge density 12 nC/m^2. Find $\boldsymbol{E}$ on both sides of the planar sheet.

ρ_s

(b) A second sheet at $z = 4$ has a uniform charge density of -12 nC/m^2. Show that $\boldsymbol{E}$ exists only between the planar sheets. Find E.

A14:

a) $$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a}_n$$

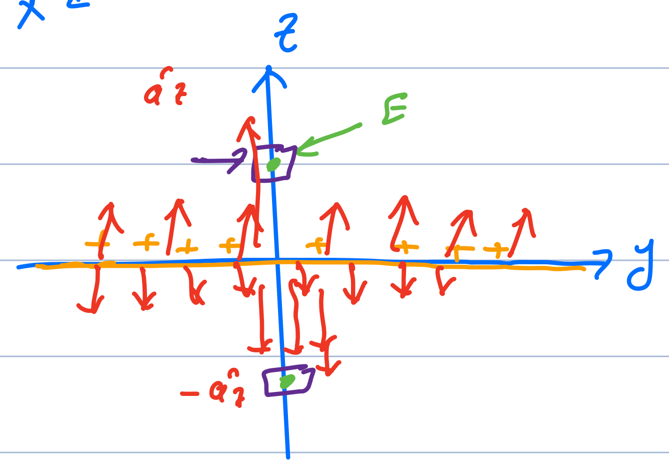


Positive side

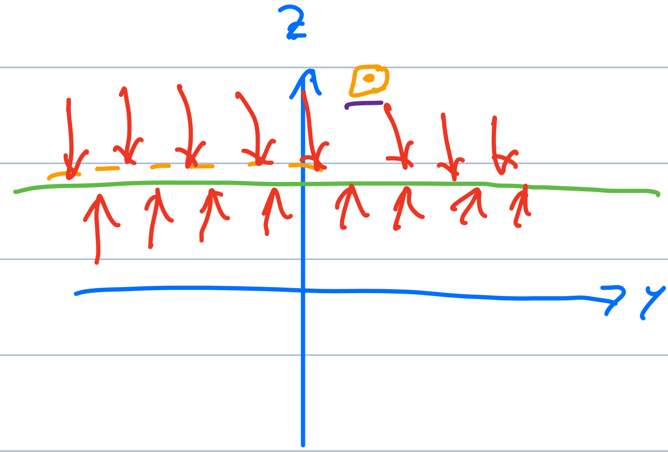
$$\vec{E} = \frac{12\times10^{-9}}{2\varepsilon_0}\,\hat{a}_z$$

negative side

$$\vec{E} = -\frac{12\times10^{-9}}{2\varepsilon_0}\,\hat{a}_z$$



b)



up side

$$\vec{E} = \frac{-12\times10^{-9}}{2\varepsilon_0}\,\hat{a}_z$$

down side

$$\vec{E} = \frac{12\times10^{-9}}{2\varepsilon_0}\,\hat{a}_z$$

Sect. 4.4: Q24:

4.24 If $\mathbf{D} = \boxed{\sin\theta\sin\phi\,\mathbf{a}_r} + \cos\theta\sin\phi\,\mathbf{a}_\theta + \cos\phi\,\mathbf{a}_\phi$ nC/m², find: (a) the charge density at $(2, 30°, 60°)$, (b) the flux through $r = 2$, $0 < \theta < 30°$, $0 < \phi < 60°$.

A24:

(a) $2 \quad 30° \quad \phi = 60°$

م.عبدالله/0501427704

$\rho_v \qquad \nabla\cdot\vec{D} = \rho_v$

D_ϕ

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \boxed{A_r}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\boxed{A_\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \boxed{A_\phi}}{\partial\phi}$$

$D_r \quad D_\theta$

$$= \boxed{\frac{1}{r^2}}\boxed{\frac{\partial}{\partial r}}\left[\boxed{r^2 \sin\theta\sin\phi}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\boxed{\sin\theta\cos\theta\sin\phi}\right]$$

$$+ \boxed{\frac{1}{r\sin\theta}}\frac{\partial}{\partial\phi}\left[\cos\phi\right]$$

$$= \frac{2}{r}\sin\theta\sin\phi + \frac{\sin\phi}{r\sin\theta}\left[\cos^2\theta - \sin^2\theta\right] + \frac{-\sin\phi}{r\sin\theta}$$

$r = 2 \qquad \theta = 30° \qquad \phi = 60°$

$\rho_v = 0$

b)

$\psi = ?? \qquad \psi = \iint \vec{D}\cdot d\vec{s} = Q_{enc}$

$ds = r^2\sin\theta\, d\theta\, d\phi$

$$= \int_0^{60°}\int_0^{30°} \sin\phi\sin\theta\, \boxed{r^2}\sin\theta\, d\theta\, d\phi$$

$$= 2\int_0^{60°}\sin\phi\, d\phi \int_0^{30°}\sin^2\theta\, d\theta = 40.6 \text{ pC}$$

Sect. 4.5/6: Q30:

4.30 In a certain region, the electric field is given by

$$\mathbf{D} = \boxed{2\rho(z+1)\cos\phi\,\mathbf{a}_\rho} - \boxed{\rho(z+1)\sin\phi\,\mathbf{a}_\phi} + \boxed{\rho^2\cos\phi\,\mathbf{a}_z}\ \mu\text{C/m}^2$$

DP , Pz

(a) Find the charge density.

(b) Calculate the total charge enclosed by the volume $0 < \rho < 2$, $0 < \phi < \pi/2$, $0 < z < 4$.

(c) Confirm Gauss's law by finding the net flux through the surface of the volume in (b).

a) ρ_v $\nabla \cdot \vec{D} = \rho_v$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

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$\rho_v = 3(z+1)\cos\phi \quad \mu C/m^3$

الاوراق خاصة بالمشتركين لا احلل نشرها

b) $Q_{net} = \iiint \rho_v \, dv$ $dv = \rho \, d\rho \, d\phi \, dz$

$$= \int_0^4 \int_0^{\pi/2} \int_0^2 3(z+1)\cos\phi \; \rho \, d\rho \, d\phi \, dz$$

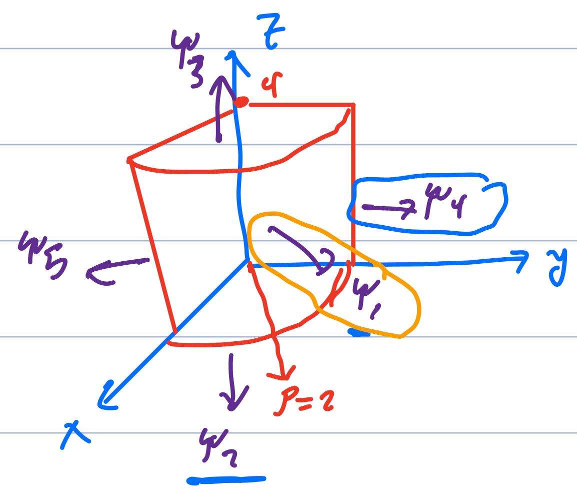
$$= 72 \ \mu C$$

c) $\Psi = \boxed{\iint \vec{D} \cdot d\vec{s}} = Q_{net}$

$\rho = 2$ $ds = \rho \, d\phi \, dz$

$$\Psi_1 = \int_0^4 \int_0^{\pi/2} 2\rho(z+1)\cos\phi \;\; \rho \, d\phi \, dz$$

$$= 96$$



$z=0 \quad ds = \rho \, d\rho \, d\phi$

$$\psi_2 = \int_0^{\pi/2}\int_0^{2} \rho^2 \cos\phi \; \rho \, d\rho \, d\phi = -4$$

$z=4 \quad ds = \rho \, d\rho \, d\phi$

$$\psi_3 = \int_0^{\pi/2}\int_0^{2} \rho^2 \cos\phi \; \rho \, d\rho \, d\phi = -4$$

$\phi = \boxed{\pi/2} \quad ds = d\rho \, dz$

$$\psi_4 = \int_0^{4}\int_0^{2} -\rho(z+1) \boxed{\sin\phi} \, d\rho \, dz = -24$$

$\phi = 0 \quad ds = d\rho \, dz$

$$\psi_5 = \int\int -\rho(z+1) \sin\phi \, d\rho \, dz = 0$$

$$\psi_T = 96 - 4 + 4 - 24 = 72 \, \mu C$$

Equations Sheet

Table - 1: Vector Identities

If **A**, **B** and **C** are vector fields, while A, B, C, U and V are scalars, and n is integer then

$\mathbf{A}\cdot\mathbf{B} = A\,B\cos\theta_{AB}$	scalar product	$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$	scalar triple product
$\mathbf{A}\times\mathbf{B} = A\,B\sin\theta_{AB}\,\boldsymbol{a}_n$	vector product	$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$	vector triple product
$A_B = A\cos\theta_{AB} = \mathbf{A}\cdot\boldsymbol{a}_B$	scalar component	$\mathbf{A}_B = A_B\boldsymbol{a}_B = (\mathbf{A}\cdot\boldsymbol{a}_B)\boldsymbol{a}_B$	vector component
$\nabla(U+V) = \nabla U + \nabla V$		$\nabla(UV) = U\,\nabla V + V\,\nabla U$	
$\nabla\left[\frac{U}{V}\right] = \frac{V\,(\nabla U) - U\,(\nabla V)}{V^2}$		$\nabla V^n = n\,V^{n-1}\,\nabla V$	
$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$		$\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\,\mathbf{B} + (\mathbf{B}\cdot\nabla)\,\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A})$	
$\nabla\cdot(V\mathbf{A}) = V\,\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla V$		$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$	
$\nabla\cdot(\nabla V) = \nabla^2 V$	div grad	$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$	
$\nabla\cdot(\nabla\times\mathbf{A}) = 0$	div curl	$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,(\nabla\cdot\mathbf{B}) - \mathbf{B}\,(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\,\mathbf{A} - (\mathbf{A}\cdot\nabla)\,\mathbf{B}$	
$\nabla\times(\nabla V) = 0$	curl grad	$\nabla\times(V\mathbf{A}) = \nabla V\times\mathbf{A} + V(\nabla\times\mathbf{A})$	
$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$	curl curl	$\oint_L V\,d\mathbf{l} = -\int_S \nabla V\times d\mathbf{S}$	
$\oint_L \mathbf{A}\cdot d\mathbf{l} = \int_S \nabla\times\mathbf{A}\cdot d\mathbf{S}$	Stokes' Theorem	$\oint_S \mathbf{A}\times d\mathbf{S} = -\int_v \nabla\times\mathbf{A}\;dv$	
$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\;dv$	Divergence Theorem	$\oint_S V\,d\mathbf{S} = \int_v \nabla V\,dv$	

Table - 2: Vector Derivatives

Cartesian (x, y, z)	Cylindrical (ρ, φ, z)	Spherical (r, θ, φ)
$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$	$\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$	$\mathbf{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$
$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$	$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$	$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$
$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$	$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$
$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla\times\mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$	$\nabla\times\mathbf{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & (r\sin\theta)\,\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & (r\sin\theta)\,A_\phi \end{vmatrix}$
$= \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\mathbf{a}_z$	$= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]\mathbf{a}_z$	$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\mathbf{a}_\phi$
$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$	$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$	$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$

Table - 3: Vector Relations

	Cartesian	Cylindrical	Spherical
Variables	x, y, z	ρ, φ, z	r, θ, φ
Vector: $\mathbf{A}$ =	$A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z$	$A_\rho\boldsymbol{a}_\rho + A_\varphi\boldsymbol{a}_\varphi + A_z\boldsymbol{a}_z$	$A_r\boldsymbol{a}_r + A_\theta\boldsymbol{a}_\theta + A_\varphi\boldsymbol{a}_\varphi$
Magnitude of vector $\lvert\mathbf{A}\rvert = A =$	$\sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$	$\sqrt{{A_\rho}^2 + {A_\varphi}^2 + {A_z}^2}$	$\sqrt{{A_r}^2 + {A_\theta}^2 + {A_\varphi}^2}$
Position vector $\overline{OP} =$	$x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z$ for $\boldsymbol{P}(x, y, z)$	$\rho\boldsymbol{a}_\rho + z\boldsymbol{a}_z$ for $\boldsymbol{P}(\rho, \varphi, z)$	$r\boldsymbol{a}_r$ for $\boldsymbol{P}(r, \theta, \varphi)$
Distance between two point d^2 =	$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$	$\rho_2^2 + \rho_1^2 - 2\rho_1\rho_2\cos(\varphi_2 - \varphi_1) + (z_2 - z_1)^2$	$r_2^2 + r_1^2 - 2r_1r_2\cos\theta_2\cos\theta_1 - 2r_1r_2\sin\theta_2\sin\theta_1\cos(\varphi_2 - \varphi_1)$
Unit vectors properties	$\boldsymbol{a}_x \cdot \boldsymbol{a}_x = \boldsymbol{a}_y \cdot \boldsymbol{a}_y = \boldsymbol{a}_z \cdot \boldsymbol{a}_z = 1$ $\boldsymbol{a}_x \cdot \boldsymbol{a}_y = \boldsymbol{a}_y \cdot \boldsymbol{a}_z = \boldsymbol{a}_z \cdot \boldsymbol{a}_x = 0$ $\boldsymbol{a}_x \times \boldsymbol{a}_y = \boldsymbol{a}_z$ $\boldsymbol{a}_y \times \boldsymbol{a}_z = \boldsymbol{a}_x$ $\boldsymbol{a}_z \times \boldsymbol{a}_x = \boldsymbol{a}_y$	$\boldsymbol{a}_\rho \cdot \boldsymbol{a}_\rho = \boldsymbol{a}_\varphi \cdot \boldsymbol{a}_\varphi = \boldsymbol{a}_z \cdot \boldsymbol{a}_z = 1$ $\boldsymbol{a}_\rho \cdot \boldsymbol{a}_\varphi = \boldsymbol{a}_\varphi \cdot \boldsymbol{a}_z = \boldsymbol{a}_z \cdot \boldsymbol{a}_\rho = 0$ $\boldsymbol{a}_\rho \times \boldsymbol{a}_\varphi = \boldsymbol{a}_z$ $\boldsymbol{a}_\varphi \times \boldsymbol{a}_z = \boldsymbol{a}_\rho$ $\boldsymbol{a}_z \times \boldsymbol{a}_\rho = \boldsymbol{a}_\varphi$	$\boldsymbol{a}_r \cdot \boldsymbol{a}_r = \boldsymbol{a}_\theta \cdot \boldsymbol{a}_\theta = \boldsymbol{a}_\varphi \cdot \boldsymbol{a}_\varphi = 1$ $\boldsymbol{a}_r \cdot \boldsymbol{a}_\theta = \boldsymbol{a}_\theta \cdot \boldsymbol{a}_\varphi = \boldsymbol{a}_\varphi \cdot \boldsymbol{a}_r = 0$ $\boldsymbol{a}_r \times \boldsymbol{a}_\theta = \boldsymbol{a}_\varphi$ $\boldsymbol{a}_\theta \times \boldsymbol{a}_\varphi = \boldsymbol{a}_r$ $\boldsymbol{a}_\varphi \times \boldsymbol{a}_r = \boldsymbol{a}_\theta$
Diff. length $d\mathbf{l}$ =	$dx\boldsymbol{a}_x + dy\boldsymbol{a}_y + dz\boldsymbol{a}_z$	$d\rho\boldsymbol{a}_\rho + \rho d\varphi\boldsymbol{a}_\varphi + dz\boldsymbol{a}_z$	$dr\boldsymbol{a}_r + rd\theta\boldsymbol{a}_\theta + r\sin\theta\, d\varphi\boldsymbol{a}_\varphi$
Diff. surface $d\mathbf{S}$ =	$dy\,dz\,\boldsymbol{a}_x$ $dx\,dz\,\boldsymbol{a}_y$ $dx\,dy\,\boldsymbol{a}_z$	$\rho\,d\varphi\,dz\,\boldsymbol{a}_\rho$ $d\rho\,dz\,\boldsymbol{a}_\varphi$ $\rho\,d\rho\,d\varphi\,\boldsymbol{a}_z$	$r^2\sin\theta\,d\theta\,d\varphi\,\boldsymbol{a}_r$ $r\sin\theta\,dr\,d\varphi\,\boldsymbol{a}_\theta$ $r\,dr\,d\theta\,\boldsymbol{a}_\varphi$
Diff. volume dv =	$dx\,dy\,dz$	$\rho\,d\rho\,d\varphi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\varphi$

Table - 4: Coordinate Transformation Relations

	Variables	Unit Vectors	Vector Components	Notes
Cartesian to Cylindrical	$\rho = \sqrt{x^2 + y^2}$ $\varphi = \tan^{-1}\frac{y}{x}$ $z = z$	$\boldsymbol{a}_\rho = \cos\varphi\,\boldsymbol{a}_x + \sin\varphi\,\boldsymbol{a}_y$ $\boldsymbol{a}_\varphi = -\sin\varphi\,\boldsymbol{a}_x + \cos\varphi\,\boldsymbol{a}_y$ $\boldsymbol{a}_z = \boldsymbol{a}_z$	$\begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$	
Cylindrical to Cartesian	$x = \rho\cos\varphi$ $y = \rho\sin\varphi$ $z = z$	$\boldsymbol{a}_x = \cos\varphi\,\boldsymbol{a}_\rho - \sin\varphi\,\boldsymbol{a}_\varphi$ $\boldsymbol{a}_y = \sin\varphi\,\boldsymbol{a}_\rho + \cos\varphi\,\boldsymbol{a}_\varphi$ $\boldsymbol{a}_z = \boldsymbol{a}_z$	$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix}$	$\cos\varphi = x/\rho = \frac{x}{\sqrt{x^2 + y^2}}$
Cartesian to Spherical	$r = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}$ $\varphi = \tan^{-1}\frac{y}{x}$	$\boldsymbol{a}_r = \sin\theta\cos\varphi\,\boldsymbol{a}_x + \sin\theta\sin\varphi\,\boldsymbol{a}_y + \cos\theta\,\boldsymbol{a}_z$ $\boldsymbol{a}_\theta = \cos\theta\cos\varphi\,\boldsymbol{a}_x + \cos\theta\sin\varphi\,\boldsymbol{a}_y - \sin\theta\,\boldsymbol{a}_z$ $\boldsymbol{a}_\varphi = -\sin\varphi\,\boldsymbol{a}_x + \cos\varphi\,\boldsymbol{a}_y$	$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$	$\sin\varphi = y/\rho = \frac{y}{\sqrt{x^2 + y^2}}$
Spherical to Cartesian	$x = r\sin\theta\cos\varphi$ $y = r\sin\theta\sin\varphi$ $z = r\cos\theta$	$\boldsymbol{a}_x = \sin\theta\cos\varphi\,\boldsymbol{a}_r + \cos\theta\cos\varphi\,\boldsymbol{a}_\theta - \sin\varphi\,\boldsymbol{a}_\varphi$ $\boldsymbol{a}_y = \sin\theta\sin\varphi\,\boldsymbol{a}_r + \cos\theta\sin\varphi\,\boldsymbol{a}_\theta + \cos\varphi\,\boldsymbol{a}_\varphi$ $\boldsymbol{a}_z = \cos\theta\,\boldsymbol{a}_r - \sin\theta\,\boldsymbol{a}_\theta$	$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix}$	$\cos\theta = z/r = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$ $\sin\theta = \rho/r = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$
Cylindrical to Spherical	$r = \sqrt{\rho^2 + z^2}$ $\theta = \tan^{-1}\frac{\rho}{z}$ $\varphi = \varphi$	$\boldsymbol{a}_r = \sin\theta\,\boldsymbol{a}_\rho + \cos\theta\,\boldsymbol{a}_z$ $\boldsymbol{a}_\theta = \cos\theta\,\boldsymbol{a}_\rho - \sin\theta\,\boldsymbol{a}_z$ $\boldsymbol{a}_\varphi = \boldsymbol{a}_\varphi$	$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix}$	
Spherical to Cylindrical	$\rho = r\sin\theta$ $\varphi = \varphi$ $z = r\cos\theta$	$\boldsymbol{a}_\rho = \sin\theta\,\boldsymbol{a}_r + \cos\theta\,\boldsymbol{a}_\theta$ $\boldsymbol{a}_\varphi = \boldsymbol{a}_\varphi$ $\boldsymbol{a}_z = \cos\theta\,\boldsymbol{a}_r - \sin\theta\,\boldsymbol{a}_\theta$	$\begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix}$	

Table - 5: Electrostatics Relations

Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12} \approx 10^{-9}/36\pi$ F/m. Permittivity of Dielectric: $\varepsilon = \varepsilon_0 \varepsilon_r$ F/m.

$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon R^2}\mathbf{a}_R$	Force (N) (point charge)	$\mathbf{F} = \frac{Q}{4\pi\varepsilon}\sum_{k=1}^{N}\frac{Q_k(\boldsymbol{r}-\boldsymbol{r}_k)}{\lvert\boldsymbol{r}-\boldsymbol{r}_k\rvert^3}$	Force (N) (superposition)
$\mathbf{E} = \frac{\mathbf{F}}{Q} = -\nabla V = \frac{Q}{4\pi\varepsilon R^2}\mathbf{a}_R$	Electric field intensity (point charge) (V/m) (N/C)	$\mathbf{E} = \frac{1}{4\pi\varepsilon}\sum_{k=1}^{N}\frac{Q_k(\boldsymbol{r}-\boldsymbol{r}_k)}{\lvert\boldsymbol{r}-\boldsymbol{r}_k\rvert^3}$	Electric field intensity (superposition) (V/m) (N/C)
$\mathbf{D} = \varepsilon\,\mathbf{E}$	Electric flux density (C/m²)	$\psi = \int_S \mathbf{D}\cdot d\mathbf{S}$	Electric flux (C)
$\psi = Q_{enc} = \oint_S \mathbf{D}\cdot d\mathbf{S}$ (integral) $\rho_v = \nabla\cdot\mathbf{D}$ (differential)	Gauss's law (first Maxwell's equation)	$W = V_L = -Q\int_A^B \mathbf{E}\cdot d\mathbf{l} = -\int_L \mathbf{E}\cdot d\mathbf{l} + C$	Total work (J) (potential energy)
$V = \frac{Q}{4\pi\varepsilon\, r} = \frac{Q}{4\pi\varepsilon\,\lvert\boldsymbol{r}-\boldsymbol{r}`\rvert} + C$	Potential at **r** due to point charge Q (centered at **r**`)	$V_{AB} = V_B - V_A = \frac{W}{Q} = -\int_L \mathbf{E}\cdot d\mathbf{l}$	Potential difference (J/C =V)
$\boxed{V(r) = \frac{\mathbf{p}\cdot(\boldsymbol{r}-\boldsymbol{r}`)}{4\pi\varepsilon\,\lvert\boldsymbol{r}-\boldsymbol{r}`\rvert^3} + C}$	Potential at **r** due to dipole with moment **p** (centered at **r**`)	$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$	Electrostatic energy (J) due to n point charges
$\boxed{V = \frac{\mathbf{p}\cdot\boldsymbol{a}_r}{4\pi\varepsilon\, r^2}}$ $P\cdot a_r = P\cos\theta$	Potential at **r** due to dipole with moment **p** (centered at origin)	$\mathbf{E} = \frac{p}{4\pi\varepsilon\, r^3}(2\cos\theta\,\boldsymbol{a}_r + \sin\theta\,\boldsymbol{a}_\theta)$	Intensity due to dipole at origin: $p = \lvert\mathbf{p}\rvert = Qd$
$W = \oint_L \mathbf{E}\cdot d\mathbf{l} = 0$ (integral) $\nabla\times\mathbf{E} = 0$ (differential)	Kirchhoff's law (second Maxwell's equation)	$W_E = \frac{1}{2}\int_v \mathbf{D}\cdot\mathbf{E}\,dv = \frac{1}{2}\int_v \varepsilon\,\lvert\mathrm{E}\rvert^2\,dv$	Electrostatic energy (J) (continuous volume charge)
$\mathbf{J} = \frac{\Delta I}{\Delta S} = \rho_v\,\mathbf{u} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\,\mathbf{E}$	Current density (A/m²) σ: conductivity (℧/m) (S/m)	$I = \frac{dQ}{dt} = \int_S \mathbf{J}\cdot d\mathbf{S}$	Current (A)
$R = \frac{V}{I} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \mathbf{J}\cdot d\mathbf{S}} = \frac{\rho_c\, l}{S}$	Resistance (uniform) Ω $\rho_c = 1/\sigma$: resistivity	$P = V\,I = I^2 R = \int_v \mathbf{E}\cdot\mathbf{J}\,dv$	Power (uniform) W
$\rho_{ps} = \mathbf{P}\cdot\boldsymbol{a}_n$ $\rho_{pv} = -\nabla\cdot\mathbf{P}$	Bound (or polarization) surface and volume charge densities	$\mathbf{P} = \chi_e\varepsilon_0\mathbf{E}$	Polarization intensity (C/m²) $\chi_e = \varepsilon_r - 1$: susceptibility
$\nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}$	Continuity equation	$T_r = \frac{\varepsilon}{\sigma}$	Relaxation time (sec)
$E_{1t} = E_{2t},\ D_{1n} - D_{2n} = \rho_s$ or $D_{1n} = D_{2n}$ (if $\rho_s = 0$)	Boundary conditions (Dielectric- Dielectric)	$D_t = \varepsilon E_t = 0,\quad D_n = \varepsilon E_n = \rho_s$	Boundary conditions (Conductor- Dielectric)
$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$	Law of refraction	$D_t = \varepsilon_0 E_t = 0,\quad D_n = \varepsilon_0 E_n = \rho_s$	Boundary conditions (Conductor- Free)
$\nabla\cdot\varepsilon\nabla V = -\rho_v$ (inhomogeneous) $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ (homogeneous)	Poisson's equation	$\nabla^2 V = 0$	Laplace's equation
$C = \frac{Q}{V} = \frac{\int_S \mathbf{D}\cdot d\mathbf{S}}{\int_L \mathbf{E}\cdot d\mathbf{l}}$	Capacitance (F)	$RC = \frac{\varepsilon}{\sigma}$	RC relation
$C = \frac{\varepsilon\, S}{d}$	Parallel-Plate capacitance (F)	$C = \frac{2\pi\varepsilon\, L}{\ln\frac{b}{a}}$	Coaxial capacitance (F)
$C = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}$	Spherical capacitance (F)	$4\pi\varepsilon\, a$	Isolated sphere capacitance (F)

	Line	Surface	Volume	Unit
Total charge $Q =$	$\int_L \rho_L\, d\boldsymbol{l}$	$\int_S \rho_S\, d\boldsymbol{S}$	$\int_v \rho_v\, dv = Q_{enc}$	C
Field intensity $\mathbf{E} =$ (finite charge distribution)	$\int_L \frac{\rho_L\, d\boldsymbol{l}}{4\pi\varepsilon\, R^2}\mathbf{a}_R$	$\int_S \frac{\rho_S\, d\boldsymbol{S}}{4\pi\varepsilon\, R^2}\mathbf{a}_R$	$\int_v \frac{\rho_v\, dv}{4\pi\varepsilon\, R^2}\mathbf{a}_R$	N/C or (V/m)
Field intensity $\mathbf{E} =$ (infinite charge distribution)	$\frac{\rho_L}{2\pi\varepsilon\,\rho}\mathbf{a}_\rho$ ρ : distance to field point	$\frac{\rho_S}{2\varepsilon}\mathbf{a}_n$	$\frac{Q}{4\pi\varepsilon\, r^2}\mathbf{a}_r$ same as of a **point** charge	
Field density $\mathbf{D} =$ (using Gauss's law)	$\frac{\rho_L}{2\pi\rho}\mathbf{a}_\rho$	$\frac{\rho_S}{2}\mathbf{a}_n$	$\begin{cases} \frac{r}{3}\rho_o\mathbf{a}_r, & 0 < r \le a \\ \frac{a^3}{3r^2}\rho_o\mathbf{a}_r, & r \ge a \end{cases}$	C/m (line) C/m² (surface) C/m³ (volume)

Sect. 4.9: Q62:

4.62 A dipole has dipole moment $p = 2\mathbf{a}_x + 6\mathbf{a}_y - 4\mathbf{a}_z$ $\boxed{\mu}$C · m. If the dipole is located in free space at (2, 3, −1), find the potential at (4, 0, 1).

A62:

Using eq. (4.81)

$$V = \frac{P \cdot |r - r'|}{4\pi\varepsilon_0 |r - r'|^3}$$

$$r - r' = (4, 0, 1) - (2, 3, -1) = (2, -3, 2)$$

$$|r - r'| = \sqrt{17}$$

$$P \cdot |r - r'| = (2, 6, -4) \cdot (2, -3, 2)$$

$$= 4 - 18 - 8 = -22$$

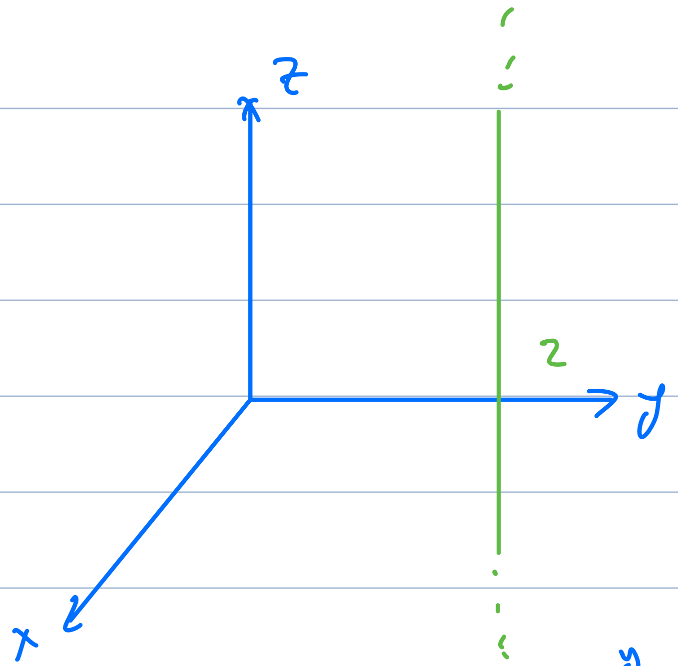
$$V = \frac{-22 \times 10^{-6}}{4\pi\varepsilon_0 \, 17^{3/2}} = -2,825 \text{ kV}$$

م.عبدالله/0501427704

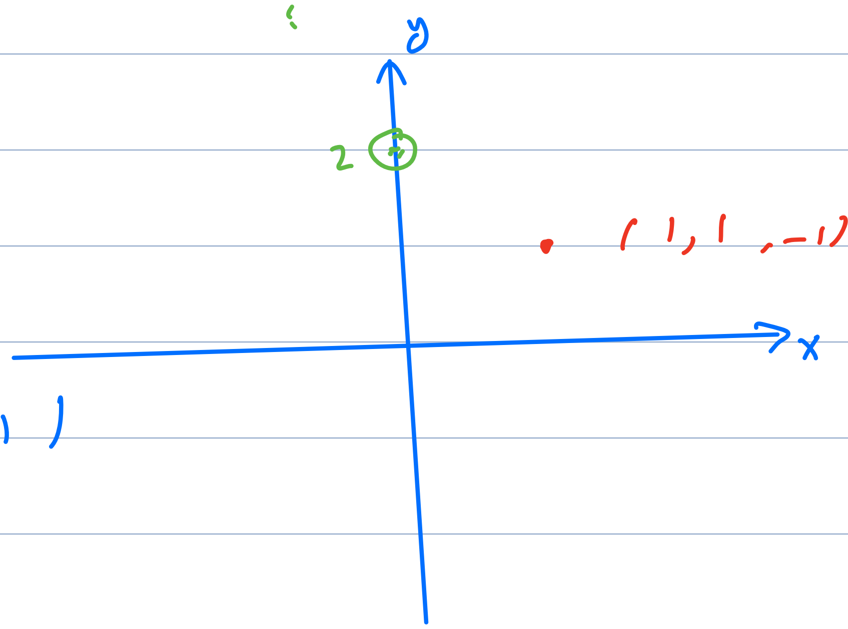
PRACTICE EXERCISE 4.6

In Example 4.6 if the line $x = 0, z = 2$ is rotated through $90°$ about the point $(0, 2, 2)$ so that it becomes $x = 0, y = 2$, find **E** at $(1, 1, -1)$.

Answer: $-282.7\mathbf{a}_x + 565.5\mathbf{a}_y$ V/m.



$$E = \frac{\rho_L}{2\pi\varepsilon_0 \boxed{\rho}} \boxed{\hat{a}_\rho}$$



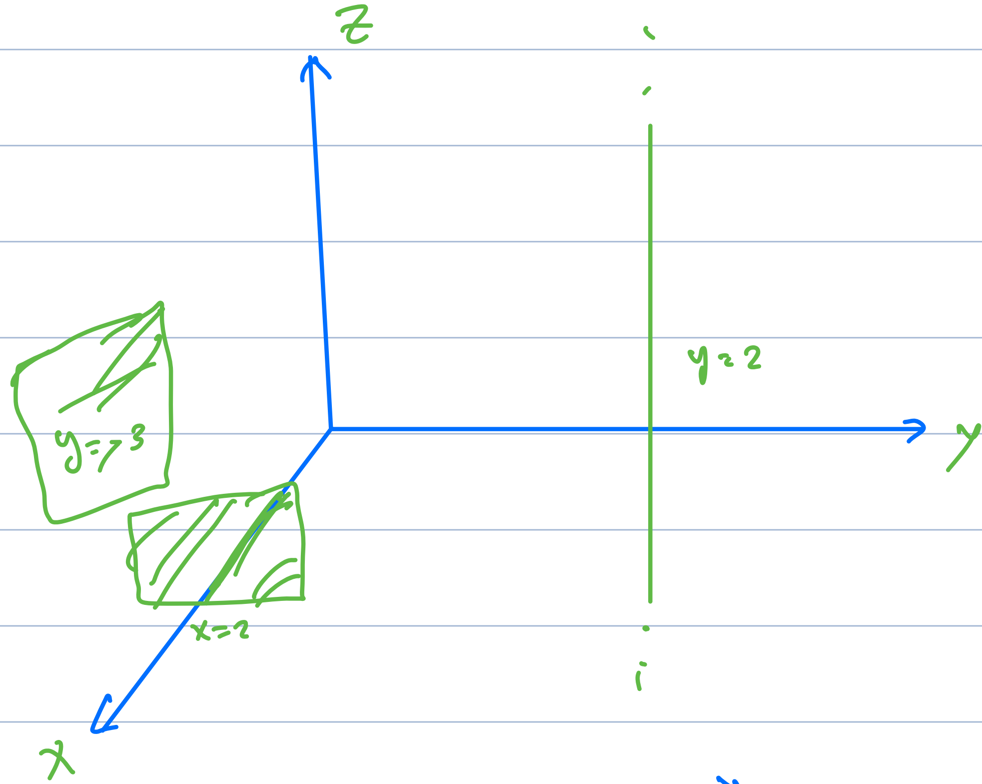
$$P = (1, 1, -1) - (0, 2, 1)$$

$$\vec{P} = (1, -1, 0)$$

$$|P| = \sqrt{2}$$

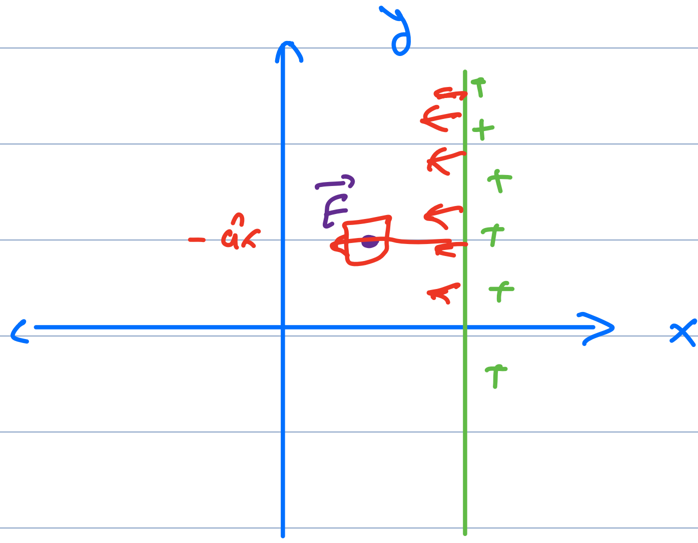
$$\hat{a}_\rho = \frac{(1, -1, 0)}{\sqrt{2}} = \frac{\hat{a}_x - \hat{a}_y}{\sqrt{2}}$$

$$\vec{E}_3 = \frac{10 \times 10^{-9}}{2\pi\varepsilon_0 \sqrt{2}} \cdot \frac{\hat{a}_x - \hat{a}_y}{\sqrt{2}}$$

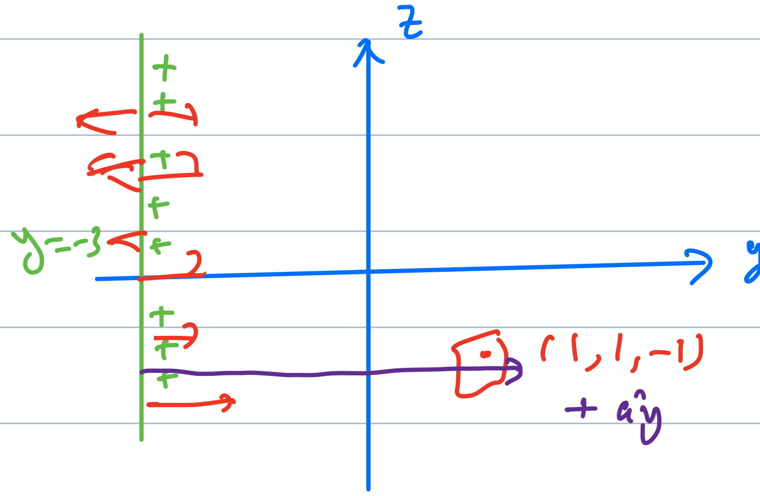


$$\vec{E}_1 = \frac{\rho_s}{2\epsilon_0}\hat{a}_r$$

$$\vec{E}_1 = \frac{10\times10^{-9}}{2\epsilon_0}(-\hat{a}_x)$$



$$\vec{E}_2 = \frac{15\times10^{-9}}{2\epsilon_0}\hat{a}_y$$



$$\vec{E}_t = \vec{E}_1+\vec{E}_2+\vec{E}_3$$

CH.4:

Coulomb's law:

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon \boxed{R^2}}\mathbf{a}_R$$

The force F between two point charges Q1 and Q2 is:

- Along the line joining them
- Directly proportional to the product Q1Q2 of the charges
- Inversely proportional to the square of the distance R between them

The electric field intensity E:

the force that a unit positive charge experiences when placed in an electric field

Gauss's Law:

$\Psi_{net} = \oiint D \cdot ds = Q_{net}$

The total electric flux through any closed surface is equal to the total charge enclosed by that surface

Electrostatic discharge (ESD):

The sudden transfer (discharge) of static charge between objects at different electrostatic potentials. ESD poses a serious threat to electronic devices and affects the operation of the systems the contains those devices.

What causes ESD? Static charge is a result of an unbalanced electrical charge at rest. For example, its created by insulator surfaces rubbing together or pulling apart. One surface gains electrons while the other loses electrons.

ESD can occur in one of four ways:

- A charged body touches a device
- A charged device touches grounded surface
- A charged machine touches a device
- An electrostatic field induces a voltage across a dielectric that Is sufficient to cause breakdown

An ESD event take place in the following four stages :
Charge generations > charge transfer > device response > device failure.

Key measurements to qualify a company ESD control program:

- Treat everything as static sensitive
- Touch something grounded before handling electronic components
- Wear a grounded wrist strap whenever possible
- Keep the relative humidity at 40% of greater
- Don't touch any leads pins or traces when handling charged devices
- Don't move around a lot

م.عبدالله/0501427704