**King Saud University**
**College of Engineering**
**Elect. Eng. Dept.**

**EE 203-Engineering Electromagnetics I**
**1st Term 1446**
**Problem Sheet I**

# Summary and Notes

**SUMMARY**

1. A field is a function that specifies a quantity in space. For example, $\mathbf{A}(x, y, z)$ is a vector field, whereas $V(x, y, z)$ is a scalar field.
2. A vector $\mathbf{A}$ is uniquely specified by its magnitude and a unit vector along it, that is, $\mathbf{A} = A\mathbf{a}_A$.
3. Multiplying two vectors $\mathbf{A}$ and $\mathbf{B}$ results in either a scalar $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$ or a vector $\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB}\, \mathbf{a}_n$. Multiplying three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ yields a scalar $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ or a vector $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$.
4. The scalar projection (or component) of vector $\mathbf{A}$ onto $\mathbf{B}$ is $A_B = \mathbf{A} \cdot \mathbf{a}_B$, whereas vector projection of $\mathbf{A}$ onto $\mathbf{B}$ is $\mathbf{A}_B = A_B\mathbf{a}_B$.
5. The MATLAB commands dot(A,B) and cross(A,B) are used for dot and cross products, respectively.

**SUMMARY**

1. The three common coordinate systems we shall use throughout the text are the Cartesian (or rectangular), the circular cylindrical, and the spherical.
2. A point $P$ is represented as $P(x, y, z)$, $P(\rho, \phi, z)$, and $P(r, \theta, \phi)$ in the Cartesian, cylindrical, and spherical systems, respectively. A vector field A is represented as $(A_x, A_y, A_z)$ or $A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$ in the Cartesian system, as $(A_\rho, A_\phi, A_z)$ or $A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$ in the cylindrical system, and as $(A_r, A_\theta, A_\phi)$ or $A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$ in the spherical system. It is preferable that mathematical operations (addition, subtraction, product, etc.) be performed in the same coordinate system. Thus, point and vector transformations should be performed whenever necessary. A summary of point and vector transformations is given in Table 2.1.
3. Fixing one space variable defines a surface; fixing two defines a line; fixing three defines a point.
4. A unit normal vector to surface $n =$ constant is $\pm\mathbf{a}_n$.

# Notes:

1- Make a good sketch of the problem. A professional engineer should be able to express the problem with appropriate engineering drawing.

2- From the sketch, try to use symmetry to simplify the analysis, whenever applicable.

3- An example of scalar fields and vector fields in Electromagnetics are:

Electric Potential V in Volt (V)

Electric field intensity **E** in V/m

We are going to investigate these fields in details in our course.

4- For a point charge Q in C, located at the origin, we have:

$$\mathbf{E} = \frac{A}{r^2}\mathbf{a}_r$$

$$V = \frac{A}{r} + Const$$

Where $A \propto Q$

5- For an infinite line on the z-axis with uniform charge density $\rho_L$ C/m, we have:

$$\mathbf{E} = \frac{B}{\rho}\mathbf{a}_\rho$$

$$V = -B\ln(\rho) + Const$$

Where $B \propto \rho_L$

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## Thinking Questions:

a) What are the units of constants A and B above?

b) Can you guess the relationship between V and **E**?

c) What should make the surfaces on which V is constant for point charge and infinite line charge cases?

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# Solved Examples:

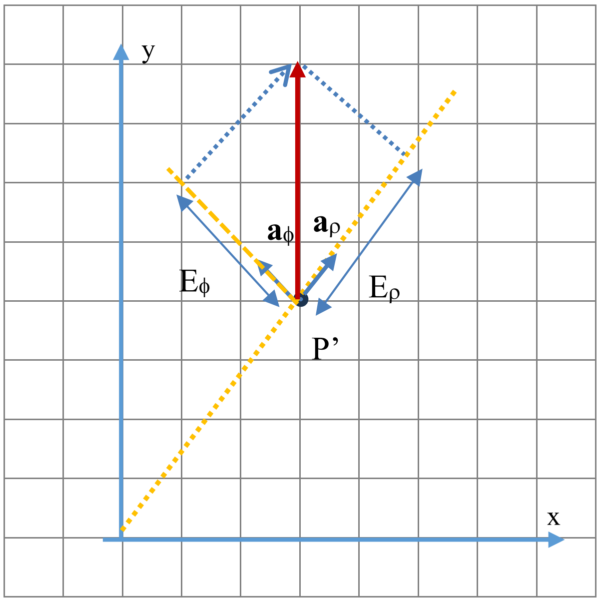
**Example 1:**

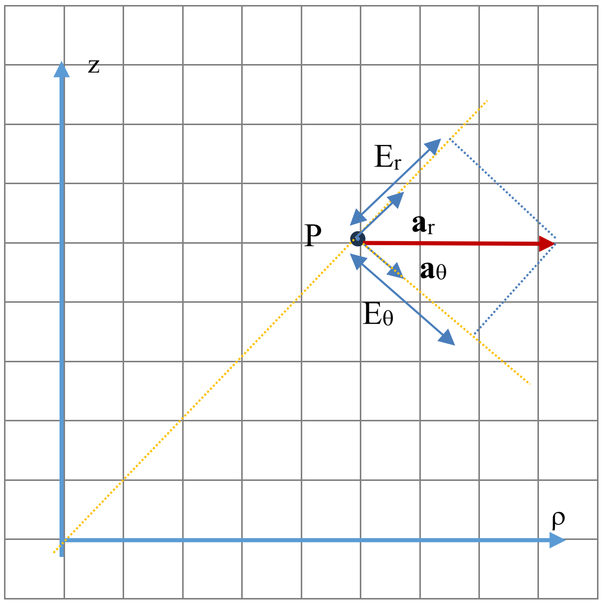
Consider the field $\vec{E} = \frac{xy}{3}\,\vec{a}_y$. Sketch the field components at P(x=3, y=4, z= 5) in x-y and ρ-z planes. Show on the figure the components of **E** in cartesian, cylindrical and spherical coordinates.

Solution:

$$\overrightarrow{E_P} = \frac{3\times 4}{3}\,\vec{a}_y = 4\,\vec{a}_y$$

Starting from project of P on x-y plane, sketch a vector of magnitude 4 in $\vec{a}_y$ direction.
Project **E** on $\vec{a}_\rho$ and $\vec{a}_\varphi$ directions.
Measure Eρ components and sketch it starting from P on ρ-z plane.
Project Eρ on $\vec{a}_r$ and $\vec{a}_\theta$ directions.

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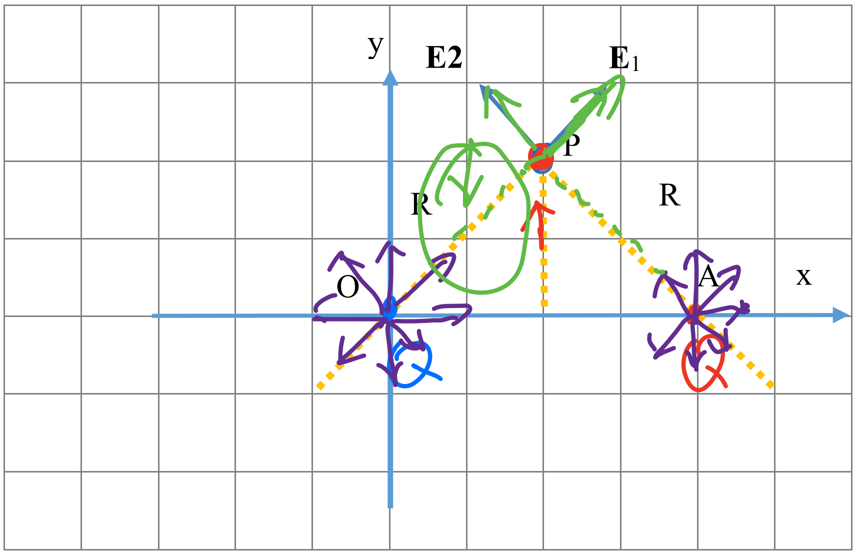
**Example 2:**

Q + postive

Using the figure below, find:

i. *If point charge Q is located at the origin, what is V and* **E***?*
ii. *If there are two charges $Q_1$=Q at O and $Q_2$=Q at A, find V and* **E** *at P(2,2,0).*




i) One charge is located at the origin.
From Note 4 above:

$$\boxed{\begin{gathered}\mathbf{E} = \frac{A}{r^2}\mathbf{a}_r \\ \mathrm{V} = \frac{A}{r} + Const\end{gathered}}$$

Where $A \propto Q$

ii) We can make use of symmetry in this case. **E** has only $\mathbf{a}_y$ component.

$$R = 2\sqrt{2}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$\mathbf{E}_1 = \frac{A}{8}\frac{(1,1,0)}{\sqrt{2}}$$

$$\mathbf{E}_2 = \frac{A}{8}\frac{(-1,1,0)}{\sqrt{2}}$$

$$\mathbf{E}_P = \frac{\sqrt{2}\,A}{8}\mathbf{a}_y \text{ V/m}$$

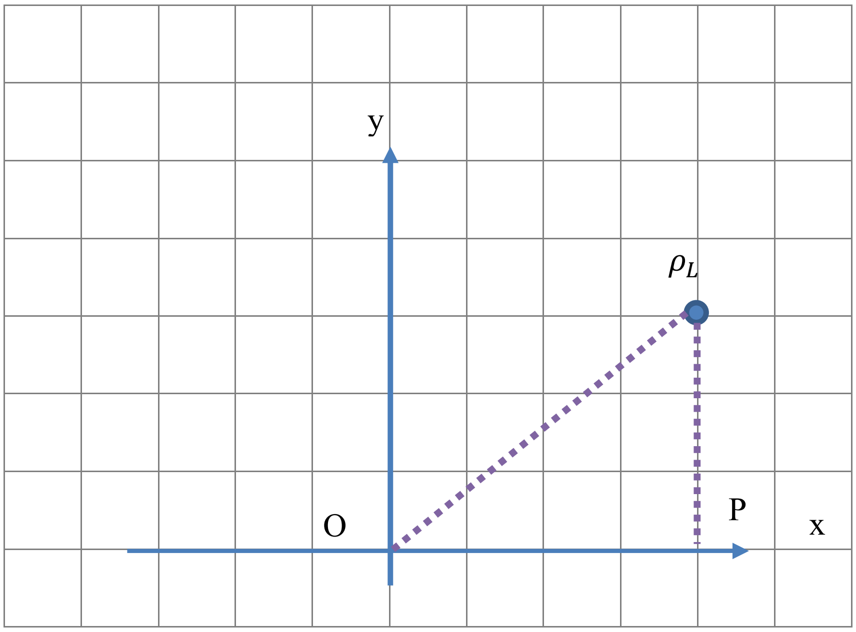
$$V_P = V_1 + V_2 = 2V_1$$

$$\mathrm{V}_P = 2\frac{A}{2\sqrt{2}} + Const$$

**Example 3:**

In the figure below, an infinite line with uniform charge density $\rho_L$ is located at x=4 and y=3. Find:

i. **E** *at O*
ii. *V at P*




Solution:

From Note 5

$$\mathbf{E} = \frac{B}{R}\mathbf{a}_R$$

For field at O, R=5 and $\mathbf{a}_R = \frac{(-4,-3,0)}{5}$

$$\mathbf{E}_O = \frac{B(-4,-3,0)}{25}$$

For V at P

$$V_P = -B\ln(3) + Const$$

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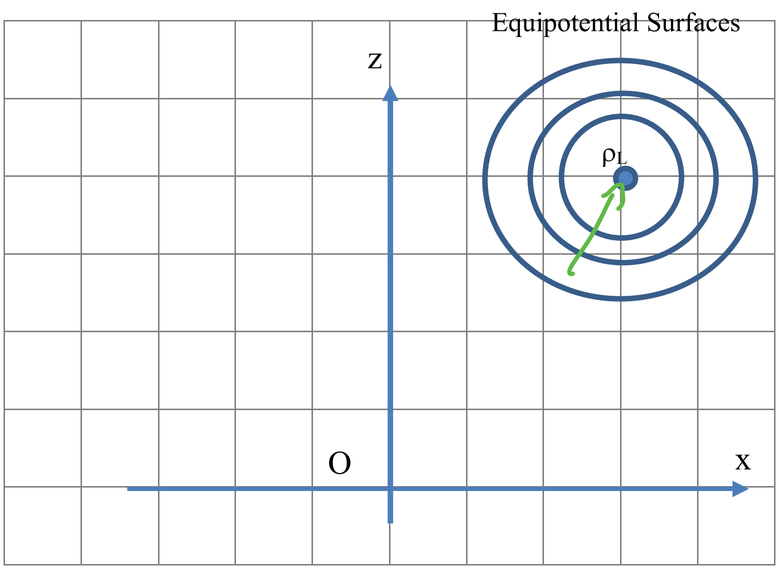
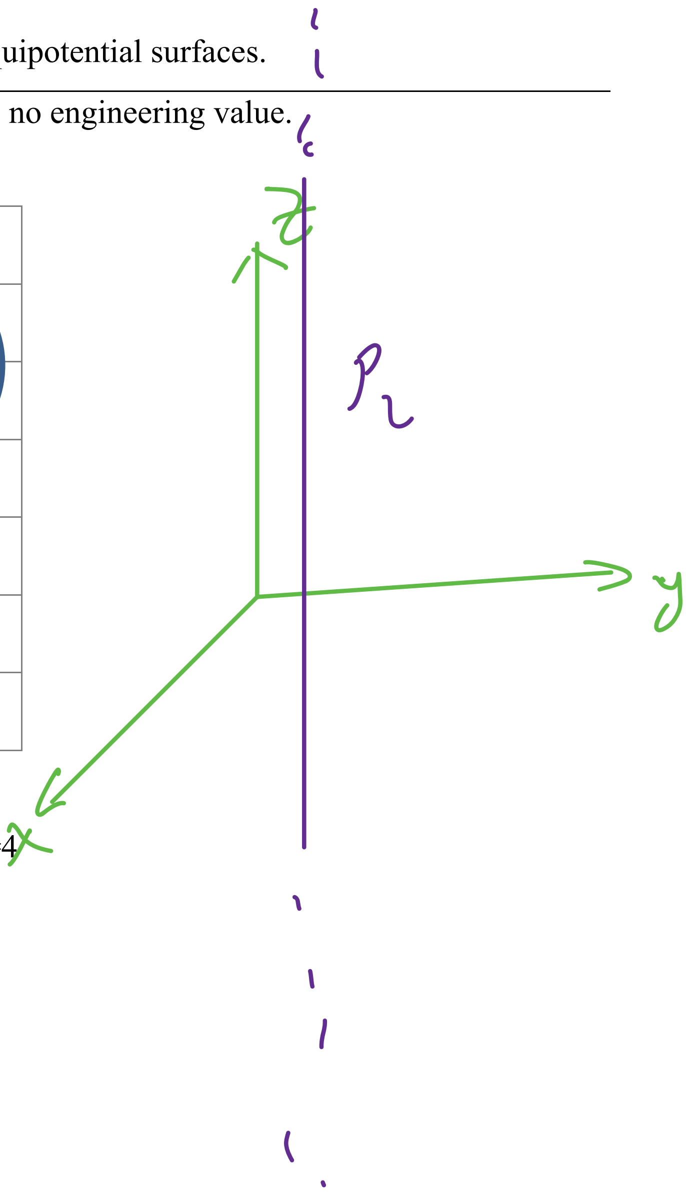
**Example 4:**

Line x=3, z=4 carries uniform charge. Identify the equipotential surfaces.

Remember: without a correct sketch, your answer has no engineering value.




Equipotential surfaces are cylinders with axis at x=3 and z=4

**Example 5:**

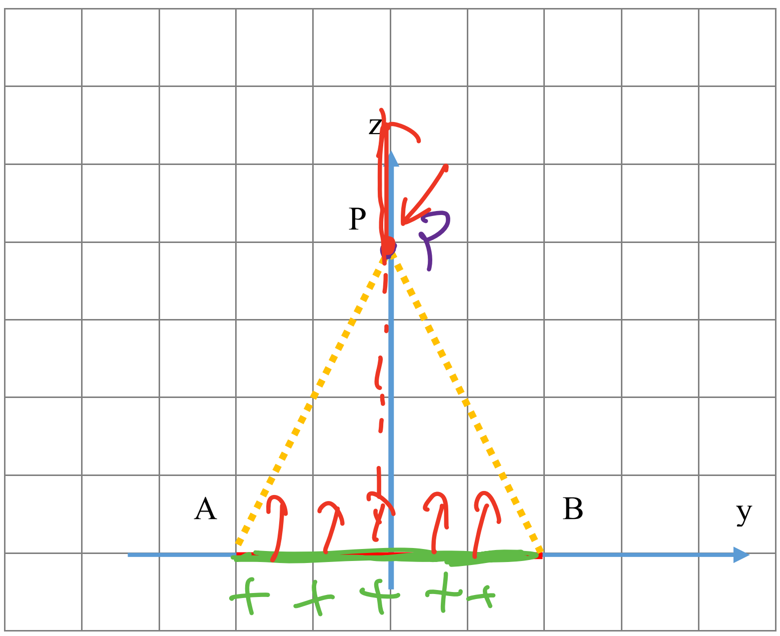
A line charge with uniform charge density exists between points A(0, -1, 0) and B(0, 1, 0). Find the direction of **E** at P(0,0,2).

Solution:

First sketch the problem.

Remember: without correct sketch, your answer has no engineering value.

From symmetry **E** should be in $\mathbf{a}_z$ direction




$\vec{E}$ at $\hat{a}_z$ direction

$\hat{z}$

$\rho = \sqrt{x^2 + y^2}$

**Example 5:**

Given that $\boxed{V = 10\rho^2 \sin\emptyset \;\; V}$, find $V$ at P(x=3, y=4, z=5)

$\rho = ?$

$\emptyset = ?$

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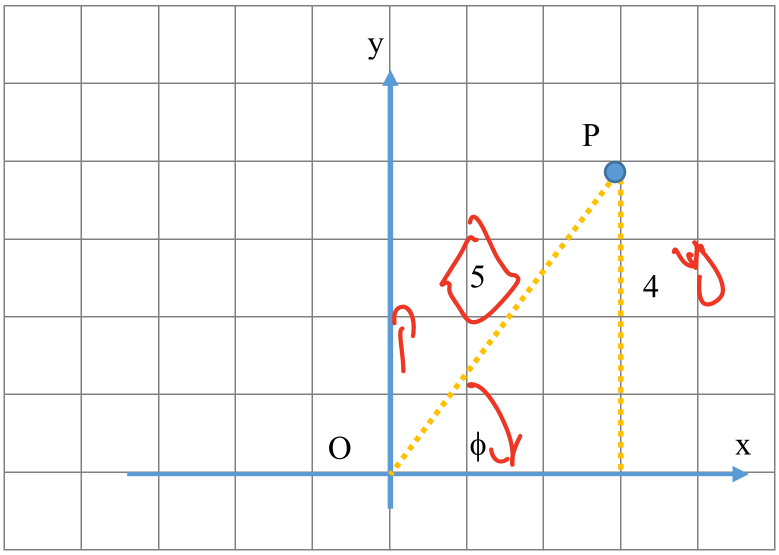
*Solution*

Make a sketch. From figure, ρ=5 and $\sin(\emptyset) = 4/5$

$$V = 10 \times 25 \times \frac{4}{5} = 200\ V$$



# Problems:

## Vector Algebra:

Q1) Correct the following statements:

i. $\vec{A} \times \vec{A} = A^2$

ii. $\vec{A} \cdot (\overrightarrow{B} \cdot \overrightarrow{C}) = \vec{B} \cdot (\overrightarrow{C} \cdot \overrightarrow{A})$

Q2) Find the vector component of $\vec{A} = 6\vec{a}_x + 2\,\vec{a}_y - 3\,\vec{a}_z$ along $\vec{B} = 3\vec{a}_x - \,\vec{a}_y$

Q3) Find the shortest distance from point P(0,0,3) to the line that passes through origin and A(2,2,2)

## Coordinate Systems:

Q1) Describe the following geometries:

i. $\theta = 30°$

ii. $r = 3 \;\; \& \; \theta = 30°$

iii. $\emptyset = \pi/2$

iv. $\rho = 5 \;\&\; \emptyset = \pi/2$

v. $x = -10$

vi. $x = -10 \;\; \& \; y = 3$

Q2) What is the unit normal vector to the cone: $\theta = 30°$?

Q3) Sketch the field $\vec{E} = xy\,\vec{a}_x + 2z\,\vec{a}_y \;\; + 2y\,\vec{a}_z$ at point P(x=0, y=1, z= 1) in x-y and ρ-z planes. Hence, express $\overrightarrow{E_P}$ in cylindrical and spherical coordinates.

Q4) Sketch the field $\vec{E} = \rho z\,\vec{a}_\rho + \rho^2\vec{a}_z$ at point P(x=0, y=-1, z= 0) in x-y and ρ-z planes. Hence, express $\overrightarrow{E_P}$ in Cartesian and spherical coordinates.

## E and V Fields (See Notes 4 and 5 abov



Q1) A charge Q is located at P(2, 3, 0).

i. Identify the equipotential surfaces.

ii. Find direction of **E** at A(2, 0, 0) and B(-, 3, 0).

Q2) An infinite line charge is located at x=4 and z=3. Find **E** *and V* at A(2*, 0, 3)*

Q3) Three charges of magnitude Q are located at A(-1, 0, 0), B(1, 0, 0) and C(3, 0, 0).

Find **E** and V at P(-3, 0, 0).

Q4) Given that $V = \frac{10}{r}\sin(\theta)\cos\emptyset$ V, find *V* at P(x=3, y=4, z=5)

$r = \sqrt{x^2 + y^2 + z^2}$

$V =$

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# Vector Algebra:

Q1) Correct the following statements:

i. $\vec{A} \times \vec{A} = A^2$

ii. $\vec{A} \cdot (\vec{B} \cdot \vec{C}) = \vec{B} \cdot (\vec{C} \cdot \vec{A})$

Q2) Find the vector component of $\vec{A} = 6\,\vec{a}_x + 2\,\vec{a}_y - 3\,\vec{a}_z$ along $\vec{B} = 3\,\vec{a}_x - \vec{a}_y$

Q3) Find the shortest distance from point P(0,0,3) to the line that passes through origin and A(2,2,2)

Q1)

i) $\vec{A} \times \vec{A} = 0$

$\vec{A} \cdot \vec{A} = A^2$

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ii)

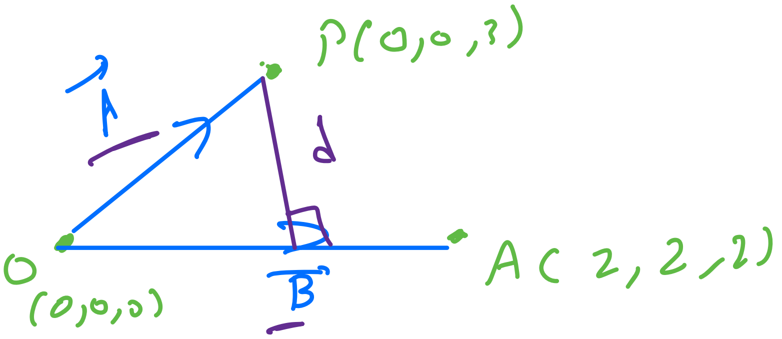
$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \cdot \vec{A})$

Q2)

$$comp_{AB} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{18 - 2 + 0}{\sqrt{9+1}} \vec{u}_B$$

$$= \frac{16}{\sqrt{10}} \vec{u}_B = \frac{16}{\sqrt{10}} \frac{3\,a_x - a_y}{\sqrt{9+1}}$$

Q3)




$$d = \frac{\vec{A} \times \vec{B}}{|\vec{B}|} = \langle -6, 6, 0 \rangle$$

$\vec{A} \times \vec{B}$

$$\begin{vmatrix} \overset{+}{\hat{a}_x} & \overset{-}{\hat{a}_y} & \overset{+}{\hat{a}_z} \\ 0 & 0 & 3 \\ 2 & 2 & 2 \end{vmatrix}$$

$= \langle -6 \ , \ 6 \ , 0 \rangle$

$= -6 \hat{a}_x + 6 \hat{a}_y$

$= \dfrac{6\sqrt{2}}{\sqrt{72}} \left[ -6 \hat{a}_x + 6 \hat{a}_y \right]$

# Coordinate Systems:

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Q1) Describe the following geometries:

i. $\theta = 30°$
ii. $\boxed{r = 3}$ & $\theta = 30°$
iii. $\emptyset = \pi/2$
iv. $\rho = 5$ & $\emptyset = \pi/2$
v. $x = -10$
vi. $x = -10$ & $y = 3$

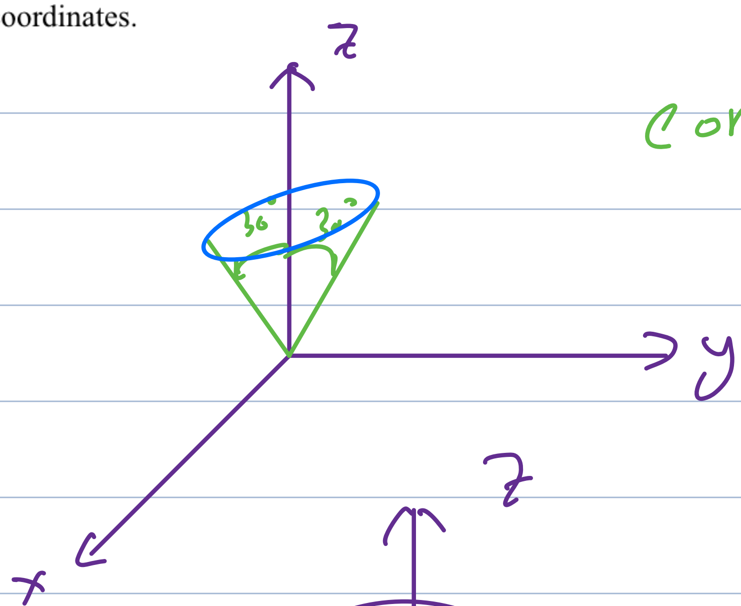
Q2) What is the unit normal vector to the cone: $\theta = 30°$?

Q3) Sketch the field $\vec{E} = xy\,\vec{a}_x + 2z\,\vec{a}_y + 2y\,\vec{a}_z$ at point P(x=0, y=1, z= 1) in x-y and ρ-z planes. Hence, express $\overrightarrow{E_P}$ in cylindrical and spherical coordinates.

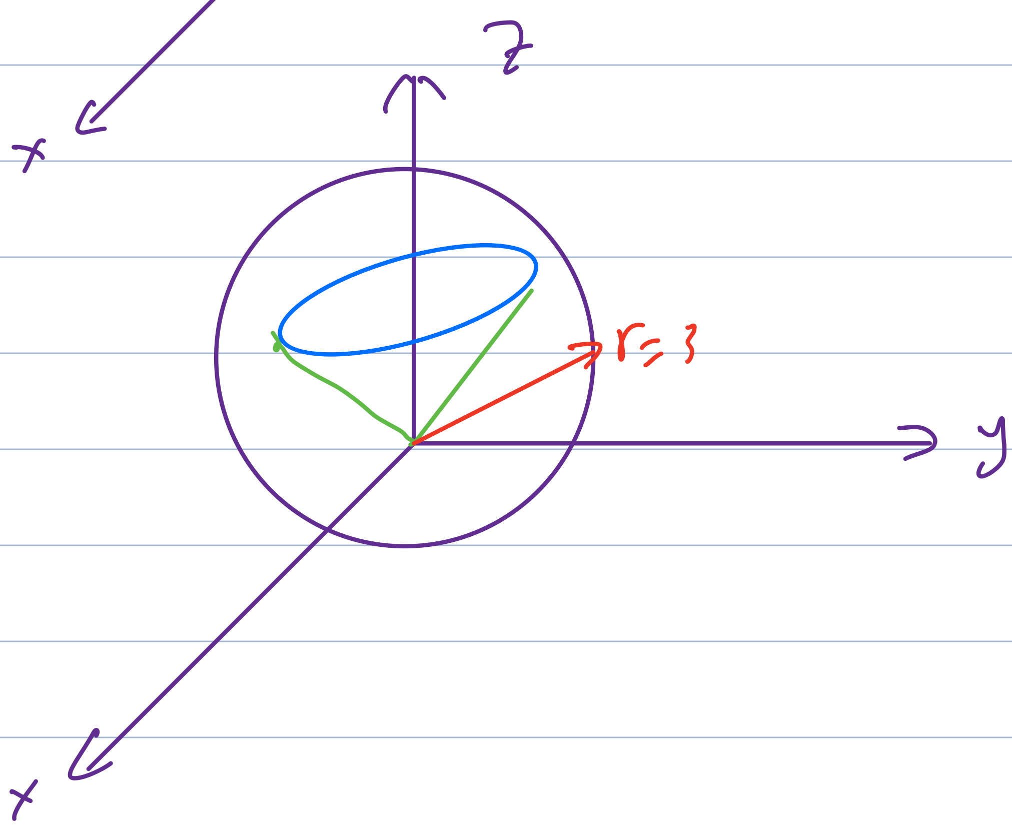
Q4) Sketch the field $\vec{E} = \rho z\,\vec{a}_\rho + \rho^2\vec{a}_z$ at point P(x=0, y=-1, z= 0) in x-y and ρ-z planes. Hence, express $\overrightarrow{E_P}$ in Cartesian and spherical coordinates.

i)

cone

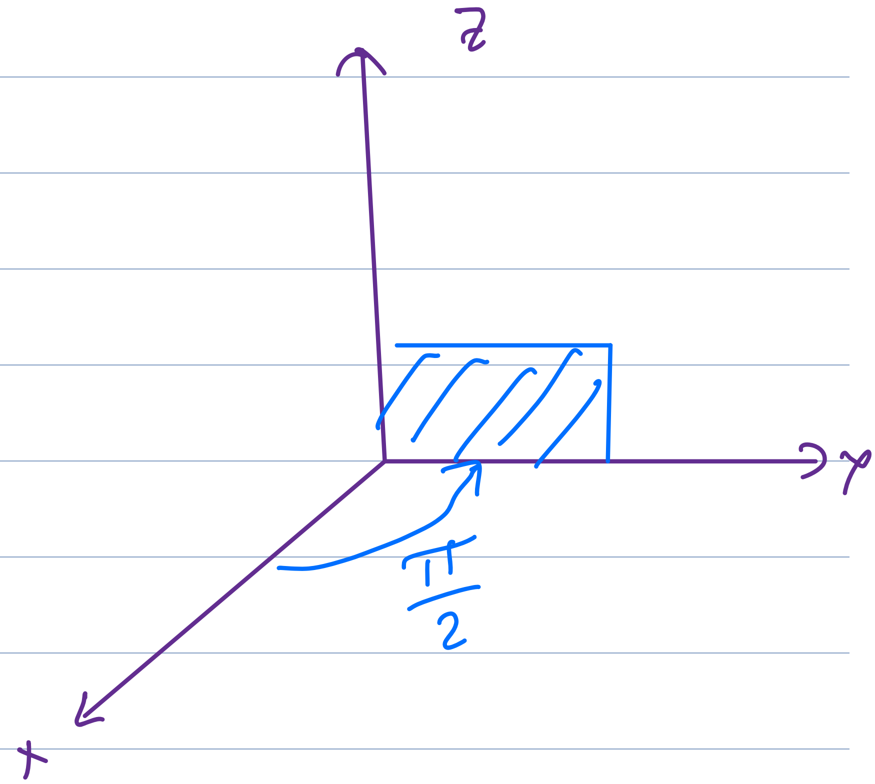



ii)



iii) plane

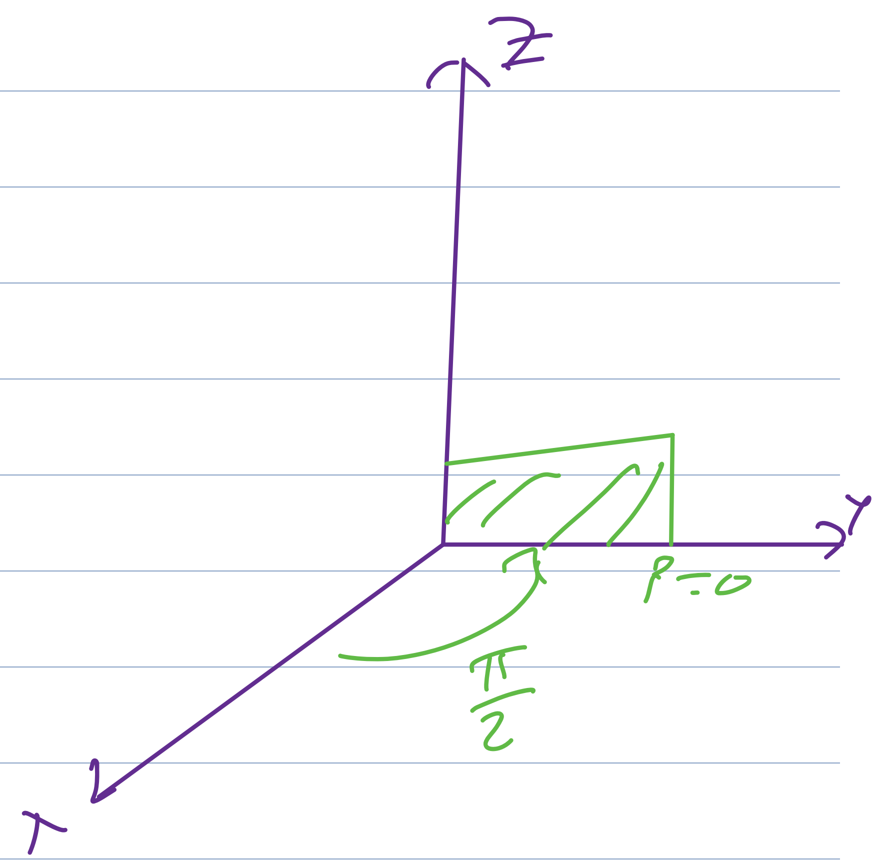
$\phi = \frac{\pi}{2}$






iv) plane

$\rho = 5 \quad \phi = \frac{\pi}{2}$



$V_f$

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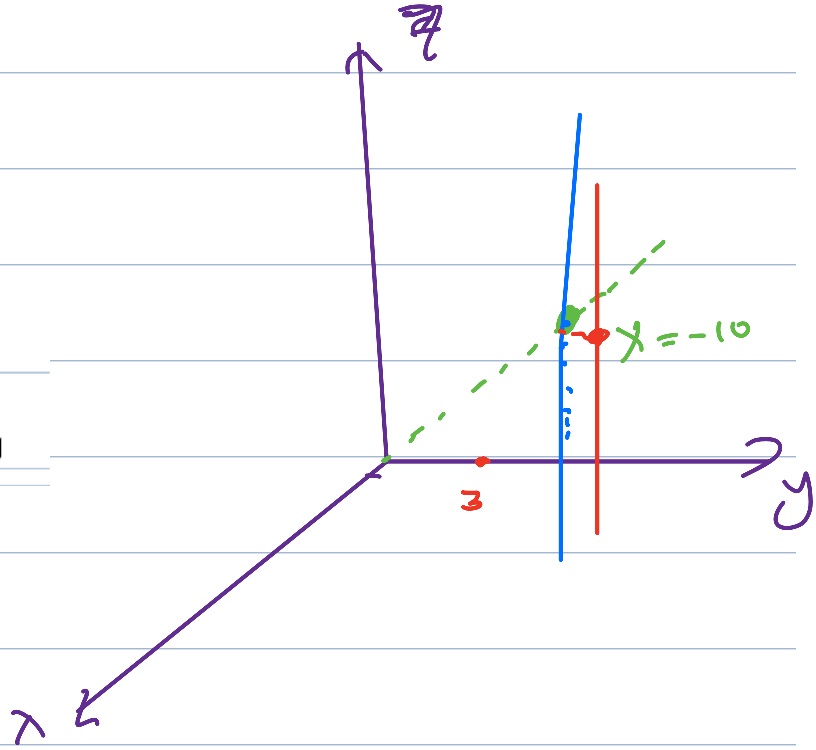

Q2) What is the unit normal vector to the cone: $\theta = 30°$?

Q3) Sketch the field $\vec{E} = xy\,\vec{a}_x + 2z\,\vec{a}_y + 2y\,\vec{a}_z$ at point P(x=0, y=1, z= 1) in x-y and ρ-z planes. Hence, express $\overrightarrow{E_P}$ in cylindrical and spherical coordinates.

Q4) Sketch the field $\boxed{\vec{E} = \rho z\,\vec{a}_\rho + \rho^2\vec{a}_z}$ at point P(x=0, y=-1, z= 0) in x-y and ρ-z planes. Hence, express $\overrightarrow{E_P}$ in Cartesian and spherical coordinates.

Q2)

$(r, \theta, \phi)$

$$\vec{N} = 1\,\hat{a}_r + 1\,\hat{a}_\phi$$

$$\vec{u}_{\hat{N}} = \frac{\vec{N}}{|\vec{N}|} = \frac{1\,\hat{a}_r + 1\,\hat{a}_\phi}{\sqrt{2}}$$

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$$= \frac{1}{\sqrt{2}}\hat{a}_r + \frac{1}{\sqrt{2}}\hat{a}_\phi$$

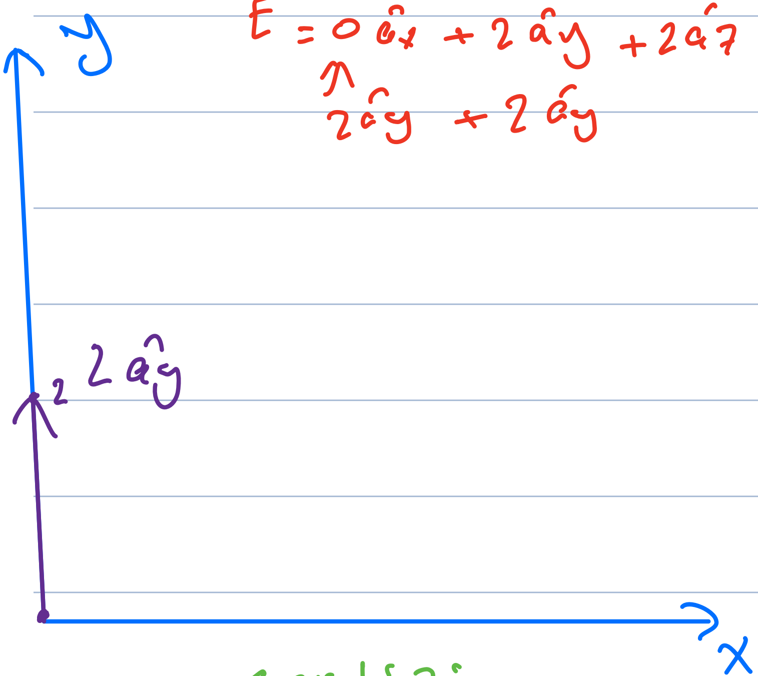
Q2)

$$\vec{E} = \underbrace{\boxed{xy\,\hat{a}_x}}_{}^{Ax} + \boxed{2z\,\hat{a}_y}^{Ay} + \boxed{2y\,\hat{a}_z}^{Az}$$

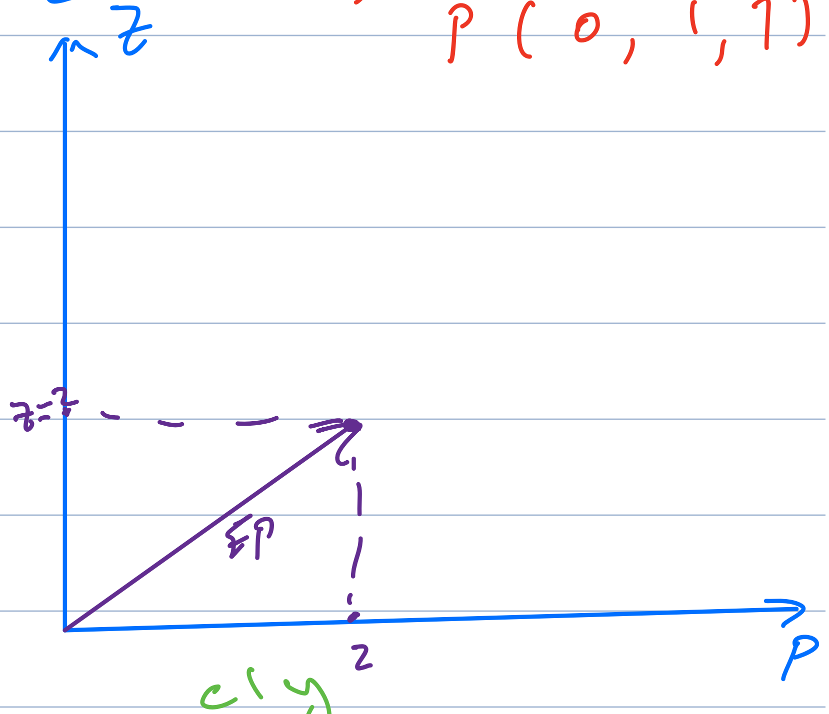
P (0, 1, 1)

$$\vec{E} = 0\,\hat{a}_x + 2\,\hat{a}_y + 2\,\hat{a}_z$$

$$2\hat{a}_y + 2\hat{a}_y$$




cartizian




cly

cart $\rightleftarrows$ cly

$P(0,1,1)$

$\phi = \tan^{-1}\frac{p}{x}$

$= \tan^{-1}\frac{2}{0}$

$= 90^\circ$

| | $A_\rho$ | $A_\phi$ | $A_z$ |
|---|---|---|---|
| $A_x$ | $\cos\phi$ | $\sin\phi$ | 0 |
| $A_y$ | $\sin\phi$ | $\cos\phi$ | 0 |
| $A_z$ | 0 | 0 | 1 |

$A_\rho = A_x \cos\phi + A_y \sin\phi + 0 A_z$

$A_x = \cos\phi \, A_\rho + A_\phi \sin\phi + 0 A_z$

$\vec{F} = 0\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z$

$A_\rho = A_x \cos\phi + A_y \sin\phi + \boxed{0 A_z}$

$A_\phi = A_x \sin\phi + A_y \cos\phi + 0 A_z$

$A_z = A_z$

$\phi = 90^\circ$

$A_\rho = 2\sin 90^\circ = 2$

$A_\phi = 0$

$A_z = 2$

$\vec{F}_p = 2\hat{a}_\rho + 0\hat{a}_\phi + 2\hat{a}_z$

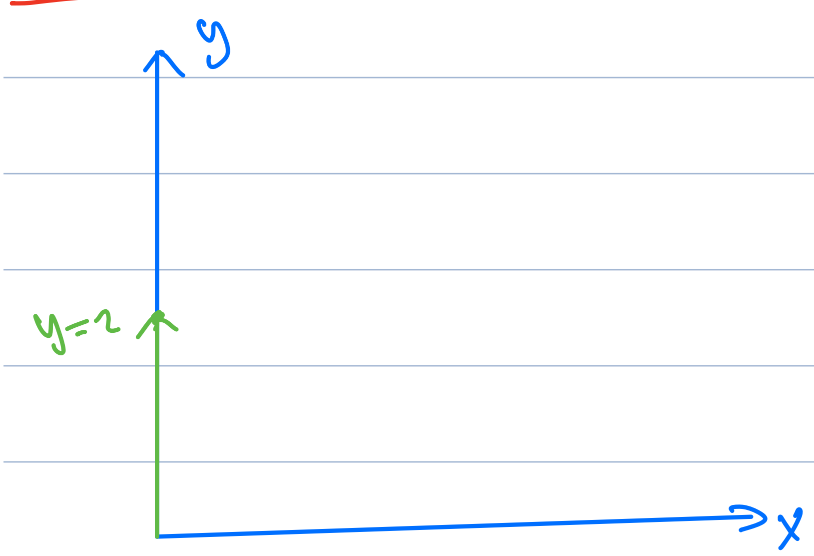
$$A_r = \frac{4}{\sqrt{2}}$$

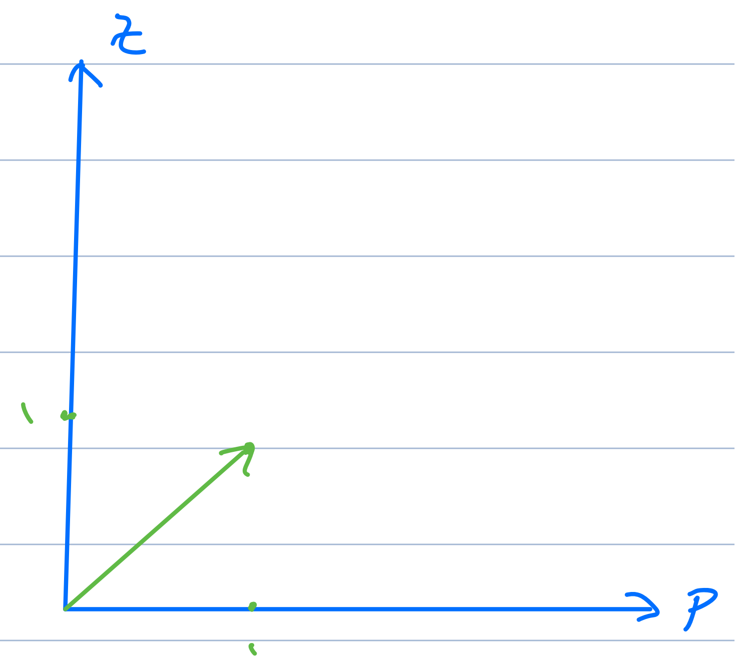
$$A_\theta = 0$$

$$A_\phi = 0$$

$$\vec{E} = \frac{4}{\sqrt{2}}\,\hat{a}_r + 0\,\hat{a}_\theta + 0\,\hat{a}_\phi$$

Q4




cart $\quad \vec{E} = 0\,\hat{a}_x + 1\,\hat{a}_y + \hat{a}_z$

cly $\quad \vec{E} = 1\,\hat{a}_\rho + 0\,\hat{a}_\phi + 1\,\hat{a}_z$

**King Saud University**
**College of Engineering**
**Electrical Engineering Department**

**EE203: Engineering Electromagnetics I**
**3rd Trimester 1444**
**Homework 2**

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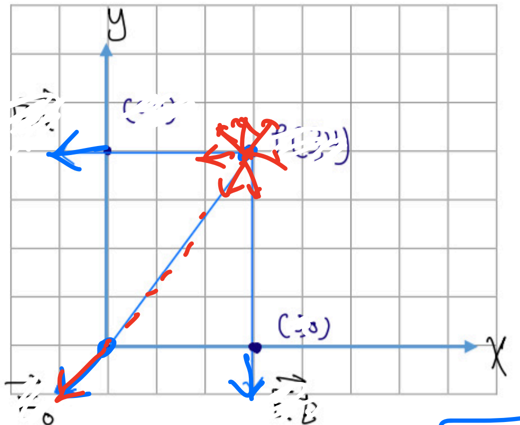
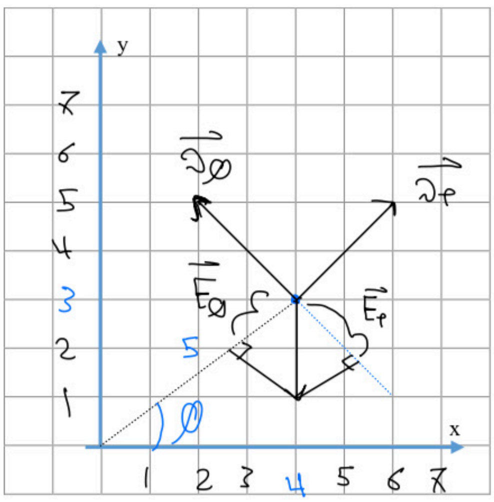
Q1) A point $\boxed{\text{positive}}$ charge Q is located at P(3, 4, 0). Makeup a sketch to show the direction of **E** at: Origin O, A(0, 4,0) and B(3, 0, 0).

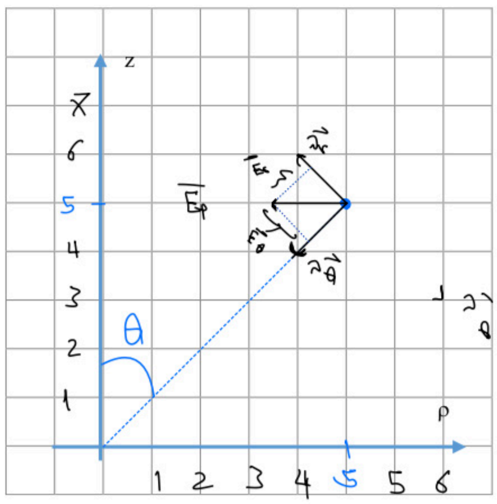


Q2) Consider the field $\vec{E} = -\frac{xy^2}{18}\vec{a}_y$. Sketch the field components at P(x=4, y=3, z= 5) in x-y and ρ-z planes. From the graph measure the following components:

$E_x$= 0 $E_y$= -2 $E_z$= 0

$E_\rho$=-2,8 $E_\phi$= -2,8 $\boxed{E_r= -2,5}$ $\boxed{E_\theta= -2,6}$





$$\vec{E} = \frac{x y^2}{18} \hat{a}_y \qquad (4, 3, 5)$$

$$\vec{E} = 0\,\hat{a}_x + \frac{x y^2}{18} \hat{a}_y + 0\,\hat{a}_z$$

$$\vec{E}_x = 0 \quad , \quad \vec{E}_z = 0 \quad , \quad \vec{E}_y = \frac{4 \times 3^2}{18}$$

$$\vec{E}_{cly} = \vec{E}_\rho + \vec{E}_\phi + \hat{E}_z$$