CH 1

"vector"

1- Vector between Two points:-

$A(x_A, y_A, z_A)$

$B(x_B, y_B, z_B)$

$\vec{AB} = B - A = \langle x_B - x_A, y_B - y_A, z_B - z_A \rangle$

2- distance between two points:-

$$|\vec{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

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3- Vector opretions :-

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\vec{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$$|\vec{A}| = \text{magnitude} = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{u}_A = \text{unite vector} : \vec{a}_A = \frac{\vec{A}}{|\vec{A}|}$$

$$\vec{A} \pm \vec{B} = (A_x \pm B_x)\,\hat{a}_x + (A_y \pm B_y)\,\hat{a}_y + (A_z \pm B_z)\,\hat{a}_z$$

- Dot product ⟶ الناتج مقدار

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$$

$\vec{A} \cdot \vec{B} = 0$ ⟵ $\theta = 90^\circ$ ⟵ perpendicular

$$\text{comp}_{AB} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$$ ← مقدار

$$\text{proj}_{AB} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}\,\vec{u}_B$$ ← متجه

$$\text{comp}_{BA} = \frac{\vec{B} \cdot \vec{A}}{|\vec{A}|}$$

$$\text{proj}_{BA} = \frac{\vec{B} \cdot \vec{A}}{|\vec{B}|\,|\vec{A}|}\,\vec{u}_A$$

$$\text{proj}_{AB} + \text{Normal comp}_{AB} = \vec{A}$$

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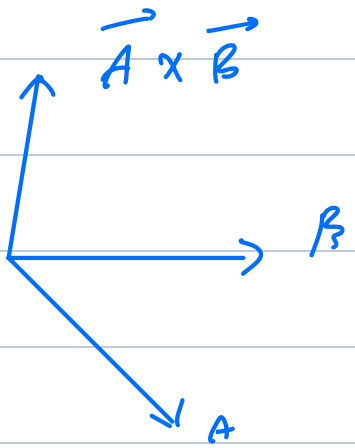
- cross - product :-



$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - B_y A_z)\hat{a}_x - (A_x B_z - B_x A_z)\hat{a}_y + (A_x B_y - A_y B_x)\hat{a}_z$$

$$\sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}|\,|\vec{B}|}$$

$\vec{A} \times \vec{B} = 0 \qquad \theta = 0$

short distance

$$d = \frac{|\vec{A} \times \vec{B}|}{|B|}$$

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CH 2

"cordinates system"

1- cordinates system :-

- car tizain :-

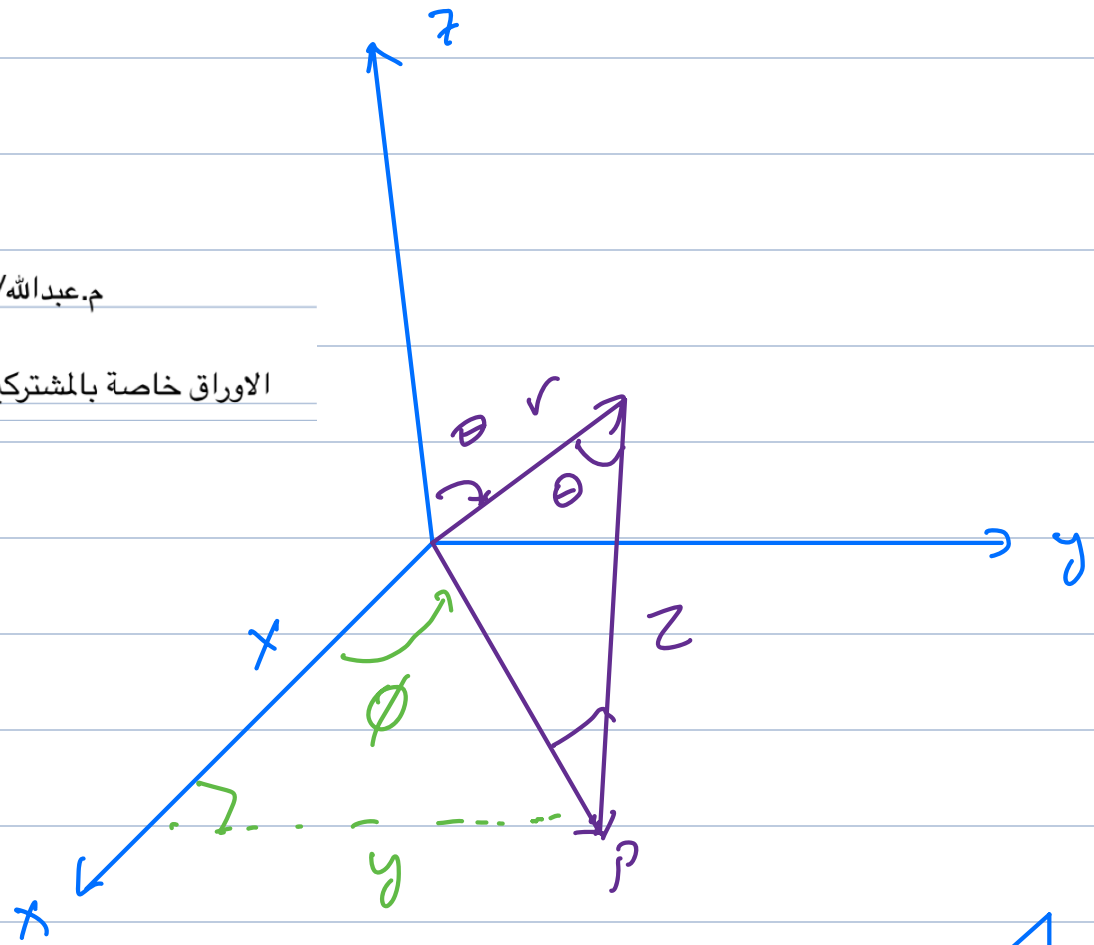
  (x, y, z)

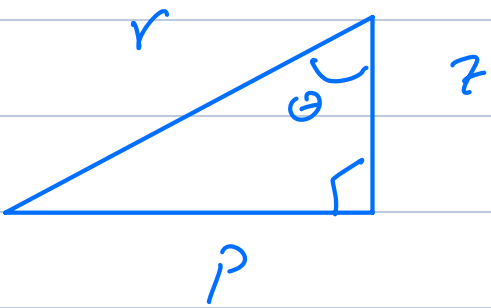
- clynider :-

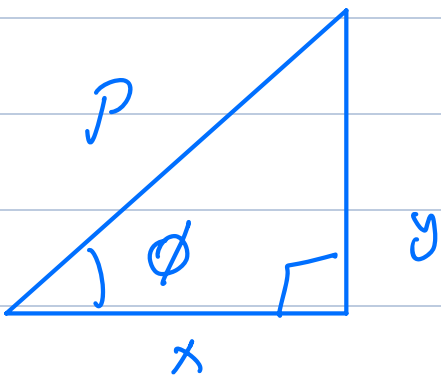
  (ρ, ϕ, z)

- sherical :-

  (r, θ, ϕ)







$$r^2 = p^2 + z^2$$

$$r^2 = x^2 + y^2 + z^2$$

$$p = r \sin\theta$$

$$z = r\cos\theta$$

$$\theta = \tan^{-1}\frac{p}{z}$$

$$x^2 + y^2 = p^2$$

$$\cos\phi = \frac{x}{p}$$

$$x = p\cos\phi$$

$$y = p\sin\phi$$

$$\phi = \tan^{-1}\frac{y}{x}$$

2- Change between cordinate:-

point ⟶ القوانين الى فوق

scaler ⟶ ؛ ؛

vector ⟶ المصفوفات الي تحت

cart ⟷ cly

	A_ρ	A_ϕ	A_z
A_x	$\cos\phi$	$\sin\phi$	0
A_y	$\sin\phi$	$\cos\phi$	0
A_z	0	0	1

$$A_\rho = A_x \cos\phi + A_y \sin\phi + 0\, A_z$$

$$A_x = \cos\phi\, A_\rho + A_\phi \sin\phi + 0\, A_z$$

cart ⟶ shp

⟵

	A_r	A_θ	A_ϕ
A_x	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
A_y	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
A_z	$\cos\theta$	$-\sin\theta$	0

cly ⟶ syn

⟵

	A_r	A_θ	A_ϕ
A_ρ	$\sin\theta$	$\cos\theta$	0
A_ϕ	0	0	1
A_z	$\cos\theta$	$-\sin\phi$	0

CH 3

"Vector Calculas"

1- length:- $\vec{dl}$

$$\vec{dl} = dx\,a_x + dy\,a_y + dz\,a_z$$

$$\vec{dl} = d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z$$

$$\vec{dl} = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$$

2- Area:-

$$\vec{ds} = dx\,dy\,a_z = dy\,dz\,a_x = dx\,dz\,a_y$$

$$\vec{ds} = \rho\,d\phi\,dz\,a_\rho = d\rho\,dz\,a_\phi = \rho\,d\phi\,d\rho\,a_z$$

$$\vec{ds} = r^2\sin\theta\,d\theta\,d\phi\,a_r = r\,dr\,d\theta\,a_\phi$$

$$= r\sin\theta\,dr\,d\phi\,a_\theta$$

3- Volume :-

$$\vec{dv} = dx\,dy\,dz$$

$$\vec{dv} = \rho\,d\rho\,d\phi\,dz$$

$$\vec{dv} = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

4- Gradiant :- ∇V ← الناتج متجه

$$\nabla V = \frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a_\rho} + \frac{1}{\rho}\cdot\frac{\partial V}{\partial \phi}\hat{a_\phi} + \frac{\partial V}{\partial z}\hat{a_z}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\cdot\frac{\partial V}{\partial \theta}\hat{a_\theta} + \frac{1}{r\sin\theta}\cdot\frac{\partial V}{\partial \phi}\hat{a_\phi}$$

Ex:-

i) $V = 8x^2yz$ ∇V

$$= 16xyz\,\hat{a_x} + 8x^2z\,\hat{a_y} + 8x^2y\,\hat{a_z}$$

ii) $V = 6\rho^2(\sin\phi)z^2$

$$\nabla V = 12\rho(\sin\phi)z^2\hat{a_\rho} + \frac{1}{\rho}6\rho^2(\cos\phi)z^2\hat{a_\rho} + 6\rho^2(\sin\phi)2z\,\hat{a_z}$$

iii) $V = 7r^2 \sin\theta \cos\phi$

$$\nabla V = 14 r \sin\theta \cos\phi \, a_r + 7r \cos\theta \cos\phi \, a_\theta + 7r(-\sin\phi) \, a_\phi$$

5- divergence Theorm :- ← الناتج scaler مقدار

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

cly, sph ← معطى في الاختبار

6- Stoke's Theorm:

$$\nabla \times \vec{A} = \begin{vmatrix} +\hat{a}_x & -\hat{a}_y & +\hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

cly ⟹ معطى في الاختبار
sph