

Q25. In a certain region for which  $\epsilon = 10\epsilon_0$ ,  $\mu = 2\mu_0$ , and  $\sigma = 0$ . If  $\mathbf{J}_d = 60 \sin(10^9 t - \beta z) \mathbf{a}_x$  mA/m<sup>2</sup>. Use Maxwell's equations to find:

a) Electric field intensity  $\mathbf{E}$   
 b) Magnetic flux density  $\mathbf{H}$   
 c) Determine Phase constant  $\beta$

Answer:

$$\mathbf{J}_d = 60 \sin(10^9 t - \beta z) \mathbf{a}_x$$

$\beta = \omega \sqrt{\mu \epsilon}$   
 $\omega = 10^9$   
 $\mu = 2\mu_0$   
 $\epsilon = 10\epsilon_0$   
 $B = 14.917$

a)  $\frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_d$

$$\mathbf{D} = \frac{-60}{10^9} \cos(10^9 t - \beta z) \mathbf{a}_x$$

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon} = \frac{-60}{10^9 (10\epsilon_0)} \cos(10^9 t - \beta z) \mathbf{a}_x \text{ mV/m}$$

b)  $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

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$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = \frac{-60 \beta}{10^9 \epsilon} \sin(10^9 t - \beta z) \mathbf{a}_y$$

$$\mathbf{B} = \frac{-60 \beta}{(10^9)^2 \epsilon} \cos(10^9 t - \beta z) \mathbf{a}_y$$

$$\mathbf{H} = \frac{-60 \beta}{(10^9)^2 \epsilon \mu} \cos(10^9 t - \beta z) \mathbf{a}_y \text{ mA/m}$$

Q15. The figure shows a 10 turn square loop centered at the origin and having 20 cm sides oriented parallel to the x- and y- axis

- a) Determine  $V_{emf}$  where the loop is located in the x-y plane, the magnetic flux density is  $\mathbf{B} = \mathbf{a}_z 0.3t$  Wb/m<sup>2</sup>, the internal resistance of the wire can be ignored.

Answer:

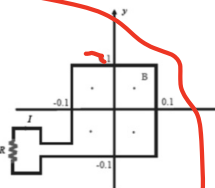
$V_{emf} =$  \_\_\_\_\_

- b) Find the current  $I$  and its direction (clockwise or counterclockwise) if  $R = 1\text{K Ohms}$ .

$I =$  \_\_\_\_\_

Direction of  $I$  \_\_\_\_\_

Explanation (Steps to find the answers):



Q14. The figure shows a 10 turn square loop centered at the origin and having 20 cm sides oriented parallel to the x- and y- axis

- a) Determine  $V_{emf}$  where the loop is located in the x-y plane, the magnetic flux density is  $\mathbf{B} = \mathbf{a}_z 0.3t$  Wb/m<sup>2</sup>, the internal resistance of the wire can be ignored.

Answer:

$V_{emf} =$  \_\_\_\_\_

- b) Find the current  $I$  and its direction (clockwise or counterclockwise) if  $R = 1\text{K Ohms}$ .

$I =$  \_\_\_\_\_

Direction of  $I$  \_\_\_\_\_

Explanation (Steps to find the answers):

$$N = 10, \quad \vec{B} = 0,3 + \mathbf{a}_z^2$$

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$$i) V_{emf} = -N \frac{d\phi}{dt}$$

$$\phi = \oint \vec{B} \cdot d\mathbf{s} = \int_{-0,1}^{0,1} \int_{-0,1}^{0,1} 0,3 + \mathbf{a}_z^2 dx dy$$

$$V_{emf} = -(10) (0,04) (0,3) = 0,12 \text{ V}$$

$$ii) I = \frac{V_{emf}}{R}$$

$$I = \frac{0,12}{1\text{K}} = 0,12 \text{ mA}$$

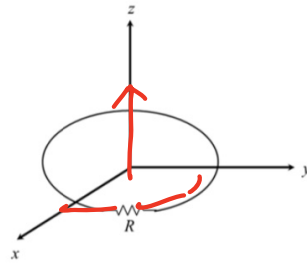
Direction of  $I$  clockwise

↑

Q22. A circuit conducting loop lies in the  $xy$ -plane as shown the Figure. The loop has the radius of 0.2 m and resistance  $R = 4 \text{ Ohms}$ . If  $\mathbf{B} = 40 \sin 10^4 t \mathbf{a}_z \text{ mWb/m}^2$ , find the current and its direction (draw it on the figure).

Answer:

$N=1$



Explanation (Steps to find the answers):

radius = 0,2 ,  $R = 4 \Omega$

$$\vec{B} = 40 \sin 10^4 t \mathbf{a}_z$$

so  $N=1$

$$\mathcal{E} = -N \frac{d\Phi}{dt}$$

$$\Phi = \int \vec{B} \cdot d\mathbf{s} = \vec{B} \cdot \mathbf{S}$$

$$S = \pi (0,2)^2$$

$$\Phi = (40 \sin 10^4 t) (\pi 0,2^2)$$

↓  
المشتق بالنسبة لـ  $t$

$$V = (-1) [40 \times 10^4 \cos(10^4 t) (\pi 0,2^2)] \text{ mV}$$

$$V = 16\pi \cos(10^4 t)$$

$$I = \frac{V}{R} = \frac{-40 \times 10^4 \times \pi \times 0,2^2 \cos(10^4 t)}{4}$$

$$I = \frac{16\pi \cos(10^4 t)}{4} , I = 4\pi \cos(10^4 t) \text{ A}$$

The  $\mathbf{a}_z$  direction Then  $I$  will be clock wise by RHL

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**Q16.** A 125-turn rectangular coil of wire with sides of 250 mm and 400 mm rotates about a horizontal axis in a vertical magnetic field of magnitude  $0.0035 \text{ Wb/m}^2$ . How fast must this coil rotate in r/s for the induced emf to reach a maximum of 1 Volts? What is the frequency in Hz?

(i) Answer:

The angular velocity is \_\_\_\_\_ r/s

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(ii) Frequency:

$f =$  \_\_\_\_\_ Hz

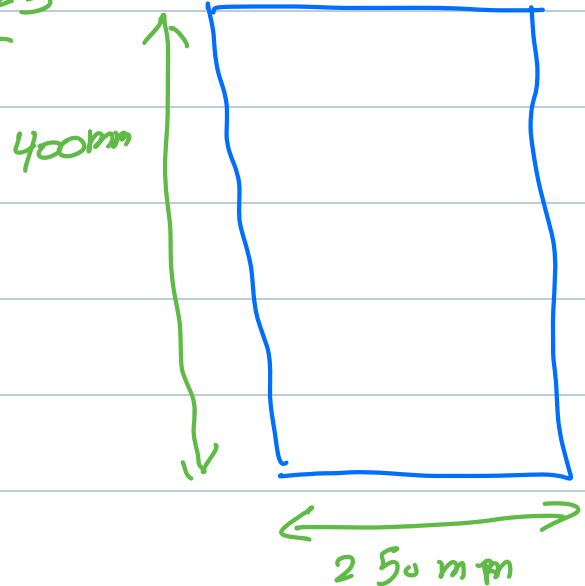
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Explanation (Steps to find the answers):

$$\vec{B} = 35 \times 10^{-4} \text{ Wb/m}^2, \quad \underline{N = 125}$$

$$V_{emf} = 1$$

$$S (\text{Area}) = 250 \text{ mm} \times 400 \text{ mm} \\ = 0,1 \text{ m}^2$$



$$\Phi = \vec{B} \cdot S$$

$$V_{emf} = -N \frac{\Delta \Phi}{\Delta t}$$

$$1 = 125 \left( \frac{35 \times 10^{-4} \times 0,1}{t} \right)$$

$$t = 0,04383$$

$$f = \frac{1}{t} = 22,857 \text{ Hz}$$

**Q24.** For a source-free region ( $\mathbf{J} = 0$ ,  $\rho_v = 0$ ,  $\epsilon = \epsilon_0$ , and  $\mu = \mu_0$ ) show that the fields  $\mathbf{E} = E_0 \cos x \cos t \mathbf{a}_y$  and  $\mathbf{H} = \frac{E_0}{\mu_0} \sin x \sin t \mathbf{a}_z$  do not satisfy all of Maxwell's equations.  
**Answer:**

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$$a) \nabla \cdot \vec{D} = \rho_v$$

$$b) \nabla \cdot \vec{B} = 0 \quad \text{Maxwell equation}$$

$$c) \nabla \times \vec{E} = - \frac{\partial B}{\partial t}$$

$$d) \nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = E_0 \sin x \cos t \mathbf{a}_z \quad (1)$$

$$\frac{\partial B}{\partial t} = \frac{E_0}{\mu_0} \sin x \cos t \mathbf{a}_z \quad (2)$$

$$-E_0 \sin x \cos t \mathbf{a}_z \neq \frac{-E_0}{\mu_0} \sin x \cos t \mathbf{a}_z$$

$$\therefore \nabla \times \vec{E} \neq - \frac{\partial B}{\partial t}$$

Then  $\vec{E}$  and  $\vec{H}$  does not satisfy Maxwell equation.

c)  $Q_3, Q_{25}$

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c)

$$\vec{D} \times \vec{H} = \vec{J}_d \quad \leftarrow$$

$$\vec{D} \times \vec{H} = \frac{60 \cdot 13^2}{(10^9)^2 \epsilon_0} \sin(10^9 t - \beta z) \hat{a}_z = \vec{J}_d$$

$$\vec{J}_d = 60 \sin(10^9 t - \beta z) \hat{a}_z$$

$$\frac{60 \beta^2}{(10^9)^2 \epsilon_0} \sin(10^9 t - \beta z) = 60 \sin(10^9 t - \beta z)$$

$$\frac{60 \beta^2}{(10^9)^2 \epsilon_0} = 60$$

$$\beta = 14, 97$$

$$= 14, 91$$

$$\approx \sqrt{\epsilon_0}$$