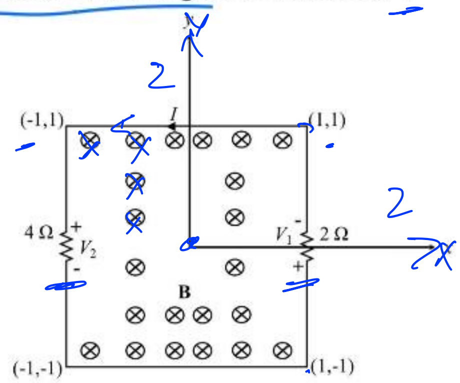
Q1) (4 Marks) The figure shows a 5 turn square loop centered at the origin and its area is 4 m^2.

If **B**= $-B_0 (x^2+1) \cos(wt)\, \mathbf{a}_z$ and $B_0 = 100\ \mu Wb/m^2$, find the following:



a) V_1 is equal to V_2. **(0.5 mark)**

1) True

2) False

b) Magnetic flux linked in the loop **(0.5 mark)**

c) Find the current I **(0.5 mark)**

d) Find the angular frequency for the induced current to reach a maximum value of -0.444 A **(0.5 mark)**

b) $\psi = \int \vec{B} \cdot ds \qquad ds = dx\, dy$

$$= \int_{-1}^{1}\int_{-1}^{1} -B_0 (x^2+1) \cos(wt)\, dx\, dy$$

$$= -B_0 \cos(wt) \int_{-1}^{1}\int_{-1}^{1} (x^2+1)\, dx\, dy$$

$$= -B_0 \cos(wt) \left[y \right]_{-1}^{1} \left[\frac{x^3}{3} + x \right]_{-1}^{1}$$

$$\psi = \frac{-16}{3} B_0 \cos(wt) \quad wb$$

c) $I = \dfrac{V_{emf}}{R}$

$R = 6 \quad , \quad V_{emf} = -N \dfrac{\partial \psi}{\partial t}$

$$\frac{\partial \psi}{\partial t} = \frac{16}{3} B_0\, w \sin(wt)$$

$$V_{emf} = -\frac{80}{3} B_0 w \sin(wt) \quad V$$

$$I = -\frac{40}{9} B_0 \boxed{w} \sin(wt) \quad A$$

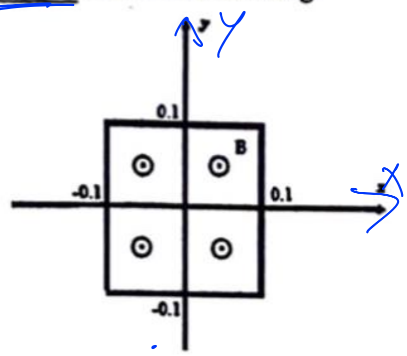
d) $-0,444 = -\frac{40}{9} B_0 w$

$$w = \frac{0,444}{\frac{40}{9} B_0} = 999 \text{ rad/s}$$

Q2 (8 Marks) Figure shows a 20 turn square loop centered at the origin and having 20 cm sides oriented parallel to the x- and y- axis. If **B**= B_0 y sin10t a_z and B_0 =10 Wb/m^2. Find the following

a) Magnetic flux linked in the loop (2 mark)

b) Induced emf Voltage. (2 mark)



Answer:

Magnetic flux = 0.04 sin 10t wb/m (2 mark)

V_{emf} = −0.08 cos(10t) V (2 mark)

Explanation (Steps to find the answers): (4 Marks)

$B = 10 y \sin 10t \, a_z$

$$\psi = \iint \vec{B} \cdot ds \qquad dx = dx\,dy$$

$$\psi = \int_{-0,1}^{0,1}\int_{-0,1}^{0,1} B_0 \, y \sin 10t \, dx\,dy$$

$$= B_0 \sin 10t \, [x]_{-0,1}^{0,1} \left[\frac{y^2}{2}\right]_{-0,1}^{0,1} = \quad wb$$

$= B_0 \sin 10t \ (0, 2)$

$\psi = 0$

$V_{emf} = 0$

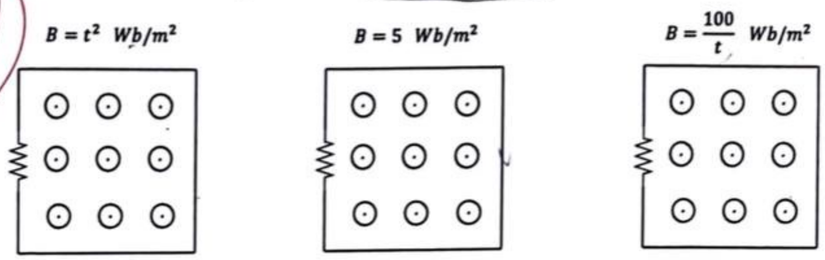
$V = -N \frac{\partial \psi}{\partial t}$

1 I

$\boxed{2}$

Q1 (12 marks) The magnetic flux density at t>0 for three different cases is shown below. For each case indicate whether a current will be generated in the wire-loop, and if it exists, specify its direction (clockwise or counterclockwise). For each case explain your reasoning briefly.

12



(a) (b) (c)

Answer:

	(a)	(b)	(c)
Generated current? (yes / no) **(3 marks)**	1 ~~No~~ Yes	~~[illegible]~~ 1 No	1 Yes
Current direction? (clockwise/ counterclockwise / -----) **(3 marks)**	clockwis 1	1 ~~clockwise~~ ~~[illegible]~~	counter clock wise 1

Explanation (Steps to find the answers): (6 marks)

1 ~~in (b) there is no~~ 2

a) yes (incresing) 2
clockwise (with right hand role)

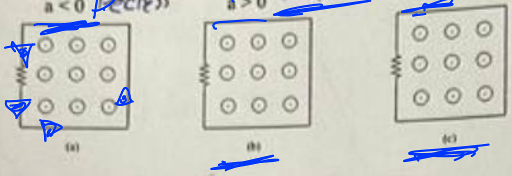
b) No (there is no component with (t))

c) Yes (decreasing) :
counter clock wise (right hand role

2

Q2) [5 Marks] The magnetic flux density at $t > 0$ for three different cases is shown below. For each case indicate whether a current will be generated in the wire-loop, and if it exists, specify its direction (clockwise or counterclockwise). For each case explain your reasoning briefly. $|\mathbf{B}| = 10t^a$ Wb/m², where **a** is a real number.

$a < 0$ Decress $\quad$ $a > 0$ increase $\quad$ $a = 0$ const



$B = 10t^3 \rightarrow a$

$B = -10t^3$

$B = 0{,}1\, t^3$

$B = 0$

Answer:

	(a)	(b)	(c)
Generated current? (yes / no) **(1.5 mark)**	Yes	Yes	No
Current direction? (clockwise/ counterclockwise / -----) **(1.5 mark)**	counter clock wise	clock wise	//

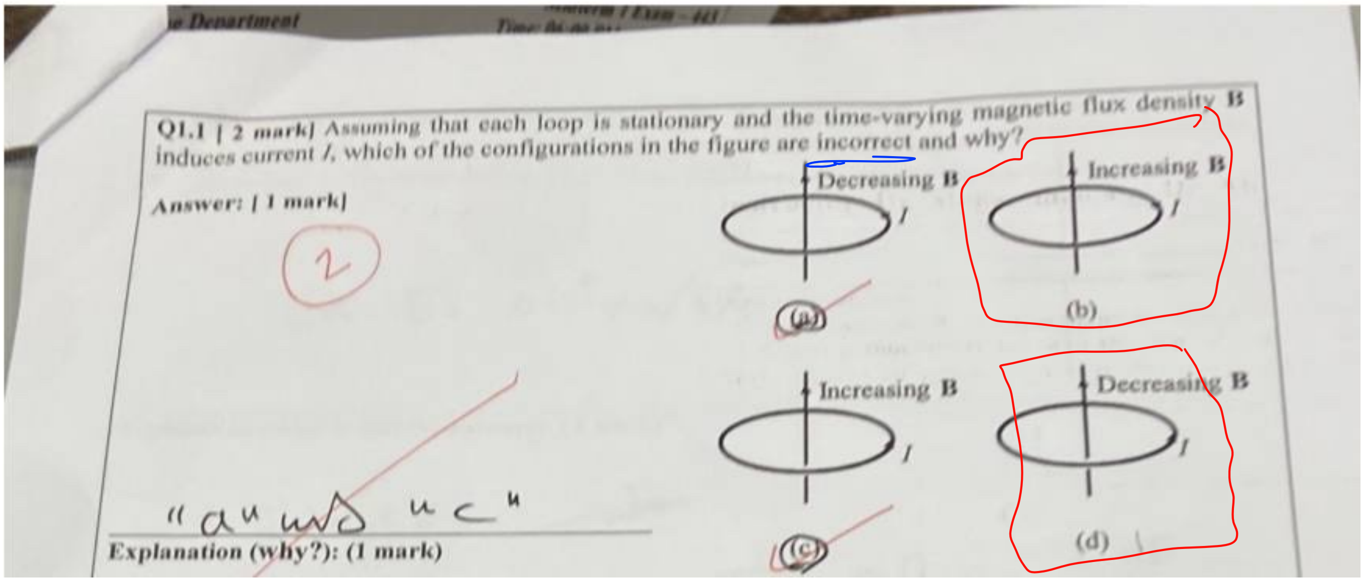
Explanation(why): [2 marks]

$V = -N \frac{d\Phi}{dt}$

2) a) B is changed with time so there is will be current, Direction we can find it using RHR $\leftarrow$ opposite $\left(-N \frac{d\Phi}{dt}\right)$

b) B is changet with time current will was ~~be~~ and by using RHR we can find the direction opposite $\left(-N \frac{d\Phi}{dt}\right)$

c) there is will be No current because B is constant



... Department

Time: ...

Q1.1 [2 mark] Assuming that each loop is stationary and the time-varying magnetic flux density B induces current I, which of the configurations in the figure are incorrect and why?

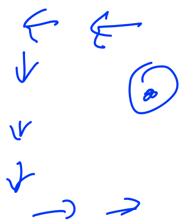
Answer: [1 mark]

2

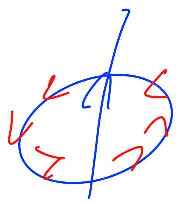
"a" and "c"

Explanation (why?): (1 mark)

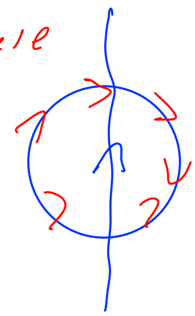
decrease



decrease



increase



a) direction incorrect

using RHL $\left(-N \frac{\partial \psi}{\partial t}\right)$

c) = = = = $-\left(N \frac{\partial \psi}{\partial t}\right)$

Q3) [5 marks]

Q3.1 (3 Marks) Write down Maxwell equations in both the integral and differential form and **briefly** explain the significance of each equation.

Eq.	Integral Form	Differential Form
1)	$\oint D \cdot ds = \int \rho_v \, dv$	$\nabla \cdot D = \rho_v$

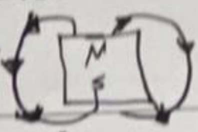
Significance/Explanation: The total flux in the surface is equal to the charge inside the surface.

⇒ Maxwell for electric field

⇒ Maxwell for magnetic field

Eq.	Integral Form	Differential Form
2)	$\oint B \cdot ds = 0$	$\nabla \cdot B = 0$

Significance/Explanation: The divergence a ~~circumstance~~ of magnetic is equal to zero and have? magnetic a circular path



Eq.	Integral Form	Differential Form
3)	$\oint E \cdot dl = -\frac{\partial}{\partial t} \int \frac{B}{ds}$	$\nabla \times E = -\frac{\partial B}{\partial t}$

Significance/Explanation: a circular loop of the electric field is equal to the magnetic field changing with time

Eq.	Integral Form	Differential Form
4)	$\oint H \cdot dl = \int J + \frac{\partial D}{\partial t} \, ds$	$\nabla \times H = \int (J + \partial D) \, ds$

Significance/Explanation: The current induced of the magnetic field circuit is equal to the J.

4) [1 mark] Identify which of the following expressions are not Maxwell's equations for time-varying fields:

1.0

$\nabla \cdot D = \rho_v$

$\nabla \cdot B$

$\nabla \times E$

$\nabla \times H$

(a) $\nabla \cdot \mathbf{J} + \frac{\partial \rho_v}{\partial t} = 0$ (✓). 0.25

(b) $\nabla \cdot \mathbf{D} = \rho_v$ ( ).

(c) $\nabla \cdot \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (✓). 0.25

(d) $\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\sigma \mathbf{E} + \frac{\partial D}{\partial t} \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) \cdot d\mathbf{S}$ ( ).

(e) $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ ( ).

Explanation(why): [0.5 mark]

a) there is no expression like that !! there is 0.25

c) $\nabla \times E = -\frac{\partial B}{\partial t}$ curl 0.25

5) [1 mark] In one dimension, a scalar wave equation takes the form of

---

Q1 (3 marks) 2.625

3.1) [1 mark] Fill in the blanks:

If a time-varying magnetic flux intercepts a surface enclosed by a closed circuit, then Lenz' law states that an induced ~~Vemf~~ current is generated such that it oppose the change of the applied magnetic flux.

0.5 ~~oppose~~ 0.5

3.2) [1 mark] The displacement current density in a medium with $\epsilon_r = 8$ and $\mu_r = 2$ is as

$$J_d = 100\,cos(10^6 t - \beta z)\,\boldsymbol{a}_x \text{ A/m}^2$$

Find the **electric field intensity** in both the phasor form **and** instantaneous form.

$D = \varepsilon \vec{E}$

$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

$\vec{J}_d = \varepsilon \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{E} = \frac{1}{\varepsilon} \int \vec{J}_d \, dt$

$\frac{1}{8\varepsilon_0}$

$\int 100 \cos(10^6 t - \beta z)\, dt = \frac{100 \sin(10^6 t - \beta z)}{10^6}$

instant. form

$\vec{E} = \frac{100}{\varepsilon_0 (8) \, 10^6} \sin(10^6 t - \beta z)\, \vec{a_x} \text{ V/m} = \boxed{1.41} \sin(10^6 t - \beta z)\, \vec{a_x} \frac{mV}{m}$ V/m 0.5

Phasor form

$\vec{E} = Im\{1.41\, e^{j\omega t} e^{-j\beta z}\, \vec{a_x}\} \frac{mV}{m} \Rightarrow \boxed{\vec{E_s} = \boxed{1.41} e^{-j\beta z}\, \vec{a_x} \frac{mV}{m}}$ 0.375

1

Fill-in the answer for each short question in the given blank.

(Provide final answers AND brief explanations in the corresponding space.)
(Note that, without explanations, you answer will not be accepted.)

Q1) [5 Marks]
Choose the correct answer and give an explanation (**note that, without explanations, you answer will not be accepted.)**

1) **[1 mark]** The polarity of V_{emf}^{tr} and hence the direction of I is governed by *Lenz's law*, which states that the current in the loop is always in a direction that opposes the magnetic flux $\varphi(t)$ that produced I.
a. True (✓) **b.** False (✓)

changing

Explanation (why):

$V_{emf}^{tr} = -N \frac{\partial \phi}{\partial t}$

opposes

$N = 100$ ϕ

2) **[1 mark]** The flux through each turn of a 100-turn coil is $(t^3\text{-}2t)$ mWb, where t is in seconds. The induced emf at $t = 2$ s is
(a) 1 V ( ). **(b)** 0.4 V ( ). **(c)** 0.01 V ( ). **(d)** −0.4 V ( ). **(e)** −1 V (✓).
Explanation(why): [0.5 mark]

$\phi = \int B.ds$ $\frac{\partial \phi}{\partial t} = 3t^2 - 2$

$V_{emf} = -N \frac{\partial \phi}{\partial t}$ $V_{emf} = (-100)(3t^2-2)\Big]_{t=2}$ $= -1V = -1$

-1000

$V = -N \frac{\partial \psi}{\partial t}$ $\frac{\partial \psi}{\partial t} = 3t^2 - 2$

3

1)

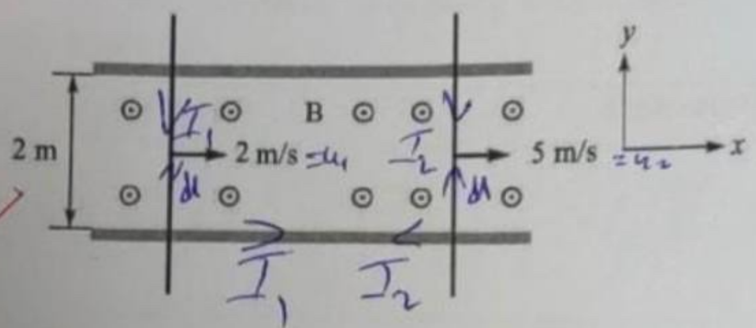
3) **[1 mark]** A loop is rotating about the y-axis in a magnetic field $\mathbf{B} = 5\, B_o \sin(wt)\, \mathbf{a}_x$ Wb/m^2. The voltage induced in the loop is due to
(a) Motional emf ( ). (b) Transformer emf ( ).
(c) A combination of motional and transformer emf (✓).
(d) None of the above ( ).
Explanation(why): [0.5 mark]

(change with time)
+ + +
loop is rotating

3

Q2 (4 Marks) Two conducting bars slide over two stationary rails, as illustrated in Figure Q2. If $B = 10\ a_z\ Wb/m^2$, determine the induced emf formed in the loop.

4



Answer:

V_{emf} = 60 V (2 mark)

Explanation (Steps to find the answers): (2 Marks)

3.5

Q3 (4 Marks)

The electric filed in vacuum is given in cylindrical coordinates as $E = 0.1 \rho z^2 e^{-4t} a_z\ V/m$, in

Find, in terms of $\mu_o \epsilon_o$:

a) The displacement current density J_D = $-0.4\, \varepsilon_0\, \rho z^2 e^{-4t}\, \vec{a_z}\ \frac{A}{m^2}$ [0.5 marks] 1

b) The magnetic field H = $0.133\, \varepsilon_0\, r^2 z^2 e^{-4t}\, \vec{a_\phi}\ A/m$ [0.5 marks] 0

c) The volume charge density in the region ρ_v = $0.2\, \varepsilon_0\, r z e^{-4t}\ C/m^3$ 1 [0.5 marks]

d) The total displacement current crossing a circular patch of a 2 m radius centered at the origin and placed at a height of 3 m. I_D = ~~$1.2\, \varepsilon_0\, e^{-4t}\ A$~~ [0.5 marks] −0.5

Explanation (Steps to find the answers): (2 marks)

$J_d = \frac{\partial D}{\ }$ $\quad \varepsilon E = D \quad D = \varepsilon_0 (0.1)\, \rho z^2 e^{-4t}\, \vec{a_z}$ ~~V/m~~

$D = \varepsilon E$

$$\vec{E} = 0{,}1\, r z^2 e^{-4t}\, a_z, \quad \vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0\, 0{,}1 \cdot r z^2 e^{-4t}\, a_z$$

a) $$J_d = \frac{\partial D}{\partial t} = -0{,}4\, \varepsilon_0\, r z^2 e^{-4t}\, a_z$$

b) $$\nabla \times \vec{H} = J_d$$

0

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(r H_\phi) - \cancel{\frac{\partial H_\rho}{\partial \phi}}\right] a_z = -0{,}4\, \varepsilon_0\, r z^2 e^{-4t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r H_\phi) = -0{,}4\, \varepsilon_0\, r z^2 e^{-4t}$$

$$\int \frac{\partial}{\partial r}(r H_\phi) = -0{,}4\, \varepsilon_0\, r^2 z^2 e^{-4t}$$

$$r H_\phi = -0{,}1333\, \varepsilon_0\, r^3 z^2 e^{-4t}$$

$$H_\phi = -0{,}1333\, \varepsilon_0\, r^2 z^2 e^{-4t}\, a_\phi\ A/m$$

1) $F_d = \int \sigma_d \, ds$

J_d

$= J_d \times S = \quad \times 2^2 \times \pi \quad = A$

...magnetics-II

2.75

Q2 [3 marks] In a certain region for which $\varepsilon = 10\varepsilon_0$, $\mu = 2\mu_0$, and $\sigma = 0$. If $\mathbf{J}_d = 60 \sin(10^9 t - \beta z)\mathbf{a}_x + 60 \sin(10^9 t - \beta z)\mathbf{a}_y$ mA/m^2

Use Maxwell's equations to find:

E a) Electric field intensity (0.5 marks) $-\frac{0.6}{10^{?}\varepsilon}\left[\cos(10^9 t - \beta z)a_x + \cos(10^9 t - \beta z)a_y\right]$ m

B b) Magnetic flux density (0.5 marks) $-\frac{\beta}{\omega\,?}\left[\cos(10^9 t - \beta z)a_y + \cos(10^9 t - \beta z)a_x\right]$ m

0.75

β c) Show that $\beta = w\sqrt{\varepsilon\mu}$ (0.5 marks) 14.91

Explanation (Steps to find the answers): (1.5 mark)

a)

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad , \quad \vec{D} = \varepsilon \vec{E} = 10\varepsilon_0 \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{10\varepsilon_0}$$

$$\vec{D} = \int J_d = \int (J_{dx} + J_{dy})$$

$$= \int J_{dx} + \int J_{dy}$$

a) $$\vec{D} = -\frac{60}{10^9}\cos(10^9 t - \beta z)\hat{a_x} - \frac{60\times10^{-3}}{10^9}\cos(10^9 t - \beta z)\hat{a_y}$$

$$\vec{E} = \frac{-60}{(10^9)(10)(\varepsilon_0)}\cos(10^9 t - \beta z)\hat{a_x} - \frac{60\times10^{-3}}{10^9(10)\varepsilon_0}\cos(10^9 t - \beta z)\hat{a_y}$$

E_x E_y

b) $\vec{B} = ??$

$$\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\vec{B} = \int \nabla \times \vec{E}$$

$$\nabla \times \vec{E} = \underline{\frac{\vec{E}_x}{\partial z}\hat{a}_y} - \underline{\frac{\vec{E}_y}{\partial z}\hat{a}_x}$$

$$\nabla \times \vec{E} = \frac{-0{,}6\beta}{10^9 \varepsilon_0}\left[\sin(10^9 t - \beta z)\hat{a}_y - \sin(10^9 t - \beta z)\hat{a}_x\right]$$

$$\vec{B} = \int \nabla \times \vec{E}$$

$$= \frac{-0{,}6\beta}{(10^9)^2 \varepsilon_0}\left[\cos(10^9 t - \beta z)\hat{a}_y - \cos(10^9 t - \beta z)\hat{a}_x\right]$$