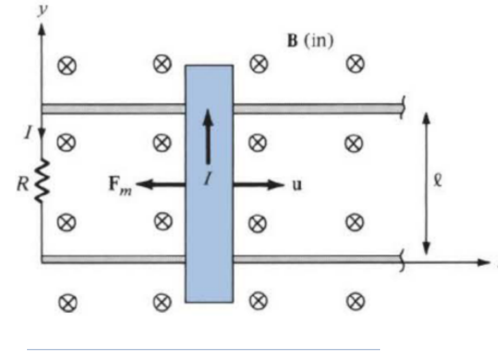
**PRACTICE EXERCISE 9.1**

Consider the loop of Figure 9.5. If $\mathbf{B} = 0.5\mathbf{a}_z$ Wb/m², $R = 20\ \Omega$, $\ell = 10$ cm, and the rod is moving with a constant velocity of $8\mathbf{a}_x$ m/s, find $= 0.1 m$

(a) The induced emf in the rod

(b) The current through the resistor $I = V/R$




1) $V_{emf} = uB\ell = 8 \times 0.5 \times 0.1 = 0.4\ V$

2) $I = \frac{V}{R} = \frac{0.4}{20} = 20 \times 10^{-3}\ A$

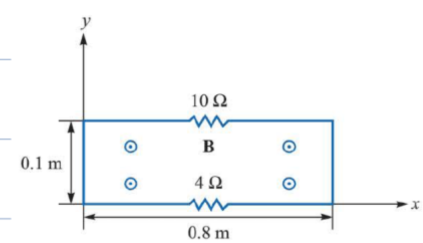
**9.1** A conducting circular loop of radius 20 cm lies in the $z = 0$ plane in a magnetic field $\mathbf{B} = 10 \cos 377t\ \mathbf{a}_z$ mWb/m². Calculate the induced voltage in the loop.

$S = \pi r^2 = \pi (0.20)^2 = \quad m^2$ , $\vec{ds} = dx\,dy\ \hat{a}_z$

$V = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$ , $\frac{\partial \vec{B}}{\partial t} = -10 \times 10^{-3} \times 377 \sin 377t\ \hat{a}_z = -3.77 \sin 377t\ \hat{a}_z$ Wb/m²

$= +3.77 \sin 377t \times \pi \times 0.20^2 = 0.4739 \sin 377t\ V$

**9.2** The circuit in Figure 9.18 exists in a magnetic field $\mathbf{B} = 40 \cos(30\pi t - 3y)\mathbf{a}_z$ mWb/m². Assume that the wires connecting the resistors have negligible resistances. Find the current in the circuit.




$ds = dx\,dy\ \hat{a}_z$

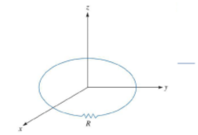
$V = -\int \frac{\partial B}{\partial t}\, ds$ , $\frac{\partial B}{\partial t} = -40 \times 30\pi \sin(30\pi t - 3y)\ \hat{a}_z$

$V = +96\pi \sin(30\pi t - 3y)$ mV

$I = \frac{V}{R}$

$I = 21.5 \sin(30\pi t - 3y)$ mA

**9.3** A circuit conducting loop lies in the *xy*-plane as shown in Figure 9.19. The loop has a radius of 0.2 m and resistance $\mathbf{R} = 4\ \Omega$. If $\mathbf{B} = 40 \sin 10^4 t\mathbf{a}_z$ mWb/m², find the currrent.



$r$

$I = \frac{V}{R}$ $\qquad$ $\vec{ds} = dx\, dy\, \hat{a}_z$

$V = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$ , $\frac{\partial B}{\partial t} = 40 \times 10^4 \cos 10^4 t\, \hat{a}_z$ Wb/m² , $S = 0,2^2 \times \pi$

$V = 400 \cos 10^4 t \times \pi \times 0,2^2 = -50,26 \cos 10^4 t$ V

$I = \frac{50,26 \cos 10^4 t}{4} = -12,56 \cos 10^4 t$ A

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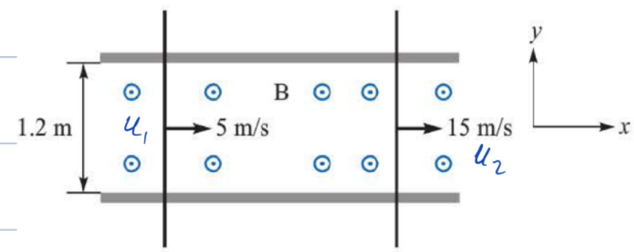
**9.4** Two conducting bars slide over two stationary rails, as illustrated in Figure 9.20. If $\mathbf{B} = 0.2\mathbf{a}_z$ Wb/m², determine the induced emf in the loop thus formed.




$V = \oint_L (\vec{u} \times \vec{B}) \cdot \vec{dl}$ $\qquad$ $u_1 \times B = 5\hat{a}_x \times 0,2\hat{a}_z = -1\hat{a}_y$

$\vec{dl} = \hat{a}_y\, dy$ $\qquad$ $u_2 \times B = 15\hat{a}_x \times 0,2\hat{a}_z = -3\hat{a}_y$

$V = \int_0^{1,2} -2\, dy$ $\qquad$ $= -3 + 1 = -2\hat{a}_y$

$= -2 \times 1,2 = -2,4$ V

**9.8** A conducting rod has one end grounded at the origin, while the other end is free to move in the $z = 0$ plane. The rod rotates at 30 rad/s in a static magnetic field $\mathbf{B} = 60\mathbf{a}_z$ mWb/m². If the rod is 8 cm long, find the voltage induced in the rod.

$r = 0,08$ m

$\omega = 30$ rad/s , $\vec{B} = 60\hat{a}_z$ mWb/m² $\qquad$ $\vec{dl} = dr\, \hat{a}_r$

$V = \cancel{-\frac{\partial}{\partial t} \int \vec{B}\, dt} = \int (\vec{u} \times \vec{B}) \cdot \vec{dl}$ $\qquad$ $\vec{u} = r\omega\, \hat{a}_\phi$

$\hat{a}_\phi \times \hat{a}_z = \hat{a}_r$

$V = \int (30r\, \hat{a}_\phi \times 60 \times 10^{-3} \hat{a}_z) \cdot dr\, \hat{a}_r = 180 \times 10^{-3} \int r\, dr$

$V = \frac{1,8}{2} r^2 \Big|_0^{0,08} \rightarrow V = 0,9(0,08^2) = 5,76 \times 10^{-3}$ V

**9.17** The ratio $J/J_d$ (conduction current density to displacement current density) is very important at high frequencies. Calculate the ratio at 1 GHz for:

(a) distilled water ($\mu = \mu_o, \varepsilon = 81\varepsilon_o, \sigma = 2 \times 10^{-3}$ S/m)

(b) seawater ($\mu = \mu_o, \varepsilon = 81\varepsilon_o, \sigma = 25$ S/m)

(c) limestone ($\mu = \mu_o, \varepsilon = 5\varepsilon_o, \sigma = 2 \times 10^{-4}$ S/m)

$J = \sigma E$ $\left\{ J_d = j\omega D \quad \text{or} = \omega D \right.$

$\omega = 2\pi f = 2\pi \times 1 \times 10^9$ rad/s

a)

$$\frac{J}{J_d} = \frac{\sigma D}{\omega D \varepsilon} = \frac{\sigma}{\omega \varepsilon} = \frac{2\times 10^{-3}}{2\pi \times 1 \times 10^9 \times 81\,\varepsilon_o} = 4{,}438 \times 10^{-4}$$

b) $$\frac{J}{J_d} = \frac{\sigma}{\omega \varepsilon} = \frac{25}{2\pi \times 1 \times 10^9 \times 81\,\varepsilon_o} = 5{,}547$$

c) $$\frac{J}{J_d} = \frac{\sigma}{\omega \varepsilon} = \frac{2\times 10^{-4}}{2\pi \times 10^9 \times 5\varepsilon_o} = 7{,}19 \times 10^{-4}$$

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**9.18** Assume that dry soil has $\sigma = 10^{-4}$ S/m, $\varepsilon = 3\varepsilon_o$, and $\mu = \mu_o$. Determine the frequency at which the ratio of the magnitudes of the conduction current density (J) and the displacement current density ($J_d$) is unity. $= 1$

$$\frac{J}{J_d} = 1 \longrightarrow \frac{J}{J_d} = \frac{\sigma}{2\pi f \times \varepsilon} = 1 \rightarrow f = \frac{\sigma}{2\pi \varepsilon} = 599{,}17 \text{ kHz}$$

**9.22** Show that fields

$$\mathbf{E} = E_o \cos x \cos t\,\mathbf{a}_y \quad \text{and} \quad \mathbf{H} = \frac{E_o}{\mu_o} \sin x \sin t\,\mathbf{a}_z$$

do not satisfy all of Maxwell's equations.

$\vec{B} = \mu \vec{H}$

$$\begin{array}{ccc} + & - & + \\ \hline a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{array}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \frac{-\mu \partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \frac{\partial A_y}{\partial x} \hat{a}_z = -E_o \sin x \cos t\, \hat{a}_z$$

$$-\vec{E}_o \sin x \cos t\, \hat{a}_z = \frac{-\mu_o \partial \vec{H}}{\partial t} \Longrightarrow \vec{H} = \frac{E_o \sin x}{\mu_o} \int \cos t\, \hat{a}_z = \frac{1}{\mu_o} E \sin x \sin t\, \hat{a}_z$$

First eq ✓

$\nabla \cdot \vec{B} = 0$

$$\nabla \cdot (\mu_o \vec{H}) = 0 \rightarrow \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z = 0 + 0 + 0 = 0$$

Second eq ✓

$\nabla \times \vec{H} = J + J_d = \cancel{J} + \frac{\partial \vec{D}}{\partial t}$ $\sigma = 0$ $\vec{D} = \varepsilon_o \vec{E}$

$$\mathbf{H} = \frac{E_o}{\mu_o} \sin x \sin t \mathbf{a}_z$$

$\vec{D} = \varepsilon_o \vec{E} \cos x \cos t \, \hat{a_y}$

$\frac{\partial \vec{D}}{\partial t} = -\varepsilon_o \vec{E} \cos x \sin t \, \hat{a_y}$

$\nabla \times \vec{H} = -\left(\frac{\partial A_z}{\partial x}\right) \hat{a_y}$

$= -\frac{E_o}{\mu_o} \cos x \sin t \, \hat{a_y}$

$$\begin{array}{ccc} + & - & + \\ \end{array}$$
$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$\neq$

eq $\underline{\underline{3}}$ not satisfy.

**9.24** In a certain region,

$$\mathbf{J} = (2y\mathbf{a}_x + xz\mathbf{a}_y + z^3\mathbf{a}_z) \sin 10^4 t \text{ A/m}$$

find $\rho_v$ if $\rho_v(x, y, 0, t) = 0$.

Continuity equation $\rightarrow$ $\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}$

$\nabla \cdot \vec{J} = 0 + 0 + 3z^2 \sin 10^4 t \, \hat{a_z}$

$\rho_v = -3z^2 \int \sin 10^4 t \, \hat{a_z} \, dt = +\frac{3z^2}{10^4} \cos 10^4 t \, \hat{a_z} + C$

At $z = 0$ $\rightarrow$ $\rho_v = 0$ $\rightarrow$ $C = 0$

So $\rho_v = \frac{3}{10^4} z^2 \cos 10^4 t \, \hat{a_z}$ c/m$^3$

**9.25** Given that $\mathbf{E} = E_o \cos(\omega t - \beta z)\mathbf{a}_x$ V/m in free space, determine **D**, **H**, and **B**.

$\varepsilon = \varepsilon_o$

$\vec{D} = \varepsilon_o \vec{E}$

$\nabla \times E = -\frac{\partial B}{\partial t}$

$H = \frac{B}{\mu_o}$

$\vec{D} = \varepsilon_o E_o \cos(\omega t - \beta z)\hat{a}_x \quad C/m^2$

$\nabla \times \vec{E} = \frac{-\partial B}{\partial t}$

$\nabla \times \vec{E} = -(0 + \beta_z E_o \sin(\omega t - \beta z)\, a_y$

$\vec{B} = +\beta E_o \int \sin(\omega t - \beta z)\, a_y\, dt$

$\vec{B} = \frac{\beta E_o}{\omega} \cos(\omega t - \beta z)\, a_y$

$\vec{H} = \frac{\beta E_o}{\mu_o \omega} \cos(\omega t - \beta z) a_y$

$$\begin{array}{ccc} + & - & + \\ \left| \begin{array}{ccc} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{array} \right| \end{array}$$

**9.26** In a certain material, $\sigma = 0$, $\mu = \mu_o$, and $\varepsilon = 81\varepsilon_o$. The magnetic field intensity in this material is $\mathbf{H} = 10 \cos(2\pi \times 10^9 t + \beta x)\mathbf{a}_z$ A/m. Determine **E** and $\beta$.

$\omega$

$$\begin{array}{ccc} + & - & + \\ \left| \begin{array}{ccc} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{array} \right| \end{array}$$

$\nabla \times \vec{H} = 0 + \frac{\partial \vec{D}}{\partial t} \longrightarrow E = \frac{1}{\varepsilon} \int \nabla \times \vec{H}\, dt$

$\nabla \times \vec{H} = +10\, \beta_x \sin(\omega t + \beta x) a_y$

$-\left(\frac{\partial A_z}{\partial x} - 0\right)$

$\vec{E} = \frac{1}{\varepsilon} \int 10\beta_x \sin(\omega t + \beta x) a_y\, dt$

$\vec{E} = \frac{-10}{\varepsilon \omega} \beta x \cos(\omega t + \beta x)\hat{a}_y \quad V/m$

$\nabla \times \vec{E} = \frac{-\partial B}{\partial t}, \quad \nabla \times \vec{E} = \frac{+10\beta^2}{\varepsilon \omega} \sin(\omega t + \beta x)\hat{a}_z$

$\vec{B} = -\frac{10\beta^2}{\varepsilon \omega} \int \sin(\omega t + \beta x)\hat{a}_z\, dt = \frac{10\beta^2}{\varepsilon \omega^2} \cos(\omega t + \beta x)\hat{a}_z$

$\vec{H} = \frac{10\beta^2}{\varepsilon \omega^2 \mu} \cos(\omega t + \beta x)\hat{a}_z$

$\vec{H} = \vec{H}$

$\frac{10\beta^2}{\varepsilon \omega^2 \mu} = 10 \longrightarrow \beta = \sqrt{81\varepsilon_o \times (2\pi \times 10^9)^2 \times \mu_o} = 188.62 \text{ rad/s}$

**9.27** In free space,

$$\mathbf{E} = \frac{50}{\rho}\cos(10^8 t - kz)\mathbf{a}_\rho \text{ V/m}$$

Find $k$, $\mathbf{J}_d$, and $\mathbf{H}$.

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\vec{D} = \varepsilon_0 \vec{E} = \frac{50\varepsilon_0}{\rho}\cos(10^8 t - Kz)\hat{a}_\rho \quad C/m^2$$

$$\nabla \times \vec{E} = \frac{50K}{\rho}\sin(10^8 t - Kz)\hat{a}_\phi$$

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\vec{H} = -\frac{50K}{\rho\mu_0}\int \sin(10^8 t - Kz)\hat{a}_\phi \, dt = \frac{50K}{\rho\mu_0 10^8}\cos(10^8 t - Kz)\hat{a}_\phi \quad A/m$$

$$\nabla \times \vec{H} = 0 + J_d \longrightarrow J_d = \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{-2.5K^2}{2\pi\rho}\sin(10^8 t - Kz)\hat{a}_\rho$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \frac{-50\varepsilon_0 \times 10^8}{\rho}\sin(10^8 t - Kz)\hat{a}_\rho \quad C/m^2$$

$$\nabla \times \vec{H} = \vec{J}_d$$

$$\frac{\cancel{-}2.5K^2}{2\pi\cancel{\rho}} = \frac{\cancel{-}50\varepsilon_0 \times 10^8}{\cancel{\rho}}$$

$$K = \sqrt{\frac{2\pi \times \varepsilon_0 \times 10^8 \times 50}{2.5}} = 0.3335$$

**9.28** The electric field intensity of a spherical wave in free space is given by

$$\mathbf{E} = \frac{10}{r} \sin\theta \cos(\omega t - \beta r)\mathbf{a}_\theta \text{ V/m}$$

Find the corresponding magnetic field intensity **H**.

$\vec{B} = \mu_o \vec{H}$

$\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)\boldsymbol{a}_\phi$

$$\nabla \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \frac{10\beta}{r} \sin\theta \sin(\omega t - \beta r)\hat{a_\phi}$$

$$\vec{H} = -\frac{1}{\mu_o}\int \frac{10\beta}{r} \sin\theta \sin(\omega t - \beta r)\hat{a_\phi}$$

$$\vec{H} = \frac{+10\beta}{\mu_o \omega r} \sin\theta \sin(\omega t - \beta r)\hat{a_\phi} \quad A/m$$

**9.29** In a certain region for which $\sigma = 0$, $\mu = 2\mu_o$, and $\varepsilon = 10\varepsilon_o$

$$\mathbf{J} = 60 \sin(10^9 t - \beta z)\mathbf{a}_x \text{ mA/m}^2$$

(a) Find **D** and **H**.
(b) Determine $\beta$.

$$\vec{D} = \int \vec{J}\, dt = \frac{-60\times10^{-3}}{10^9} \cos(10^9 t - \beta z)\hat{a_x} \quad C/m^2$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_o 10} = \frac{-60\times10^{-3}}{10^9 \times 10\varepsilon_o} \cos(10^9 t - \beta z)\hat{a_x} \quad V/m$$

$$\nabla \times \vec{E} = -\frac{\mu_o \partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \frac{60\times10^{-3}\beta \sin(10^9 t - \beta z)}{10^9 \times 10\varepsilon_o}\hat{a_y}$$

$$\vec{H} = \int \nabla \times \vec{E}\, dt = \frac{60\times10^{-3}\times\beta \cos(10^9 t - \beta z)}{10^9 \times 10\varepsilon_o \times 10^9 \times 2\mu_o}\hat{a_y}$$

$$\nabla \times \vec{H} = 0 + \vec{J_d} \longrightarrow \nabla \times \vec{H} = \frac{60\times10^{-22}\times\beta^2 \sin(10^9 t - \beta z)}{\varepsilon_o \times 2\mu_o}\hat{a_x}$$

$$\frac{\beta^2\, 60\times10^{-22}}{\varepsilon_o \times 2\mu_o} = 60\times10^{-3} \longrightarrow \beta = \sqrt{\frac{\varepsilon_o \times 2\mu_o \times 10^{-3}}{10^{-22}}} = 14.917$$

$\varepsilon_0$ $\quad \sigma = 0 \rightarrow J = 0$

**9.33** An antenna radiates in free space and

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\mathbf{H} = \frac{12 \sin\theta}{r} \cos(2\pi \times 10^8 t - \beta r)\mathbf{a}_\theta \text{ mA/m}$$

Find the corresponding **E** in terms of $\beta$.

$$\boxed{\omega = 2\pi \times 10^8}$$

$$\nabla \times \vec{H} = 0 + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \longrightarrow \vec{E} = \frac{1}{\varepsilon_0} \int \nabla \times \vec{H}\, dt$$

$$\nabla \times \vec{H} = 0 + \frac{1}{r} \frac{\partial (r H_\theta)}{\partial r} \hat{a}_\phi = \frac{12 \sin\theta}{r} \beta \sin(\omega t - \beta r)\, \hat{a}_\phi$$

$$\vec{E} = \frac{1}{\varepsilon_0} \frac{12 \sin\theta}{r} \beta \int \sin(\omega t - \beta r)\, \hat{a}_\phi\, dt = \frac{-12 \sin\theta}{\varepsilon_0 \times r \times \omega} \beta \cos(\omega t - \beta r)\, \hat{a}_\phi \quad \text{mV/m}$$

$$\vec{E} = \frac{-12 \sin\theta}{\varepsilon_0 r (2\pi \times 10^8)} \beta \cos(2\pi \times 10^8 t - \beta r)\, \hat{a}_\phi \quad \text{mV/m}$$